PREFERENCE DISAGGREGATION FOR MEASURING AND ANALYSING CUSTOMER SATISFACTION: THE MUSA METHOD

E. Grigoroudis and Y. Siskos

Technical University of Crete
Decision Support Systems Laboratory
University Campus, 73100 Chania, GREECE
Tel. +30-821-64186 / Fax +30-821-64824
Email: dsslab@ergasya.tuc.gr  http://www.ergasya.tuc.gr

Abstract

The multicriteria method MUSA (MUlticriteria Satisfaction Analysis) for measuring and analysing customer satisfaction is presented in this paper. The MUSA method is a preference disaggregation model following the principles of ordinal regression analysis (inference procedure). The integrated methodology evaluates the satisfaction level of a set of individuals (customers, employees, etc.) based on their values and expressed preferences. Using satisfaction survey’s data, the MUSA method aggregates the different preferences in unique satisfaction functions. This aggregation-disaggregation process is achieved with the minimum possible errors. The main advantage of the MUSA method is that it fully considers the qualitative form of customers’ judgements and preferences. The development of a set of quantitative indices and perceptual maps makes possible the provision of an effective support for the satisfaction evaluation problem. The paper also presents the reliability analysis of the provided results, along with a simple numerical example that demonstrates the implementation process of the MUSA method. Finally, several extensions and future research in the context of the presented method are discussed.

Key words: Multicriteria analysis, preference disaggregation, ordinal regression, customer satisfaction analysis

1. INTRODUCTION

Customer satisfaction is one of the most important issues concerning business organisations of all types, which is justified by the customer-orientation philosophy and the main principles of continuous improvement of modern enterprises. For this reason, customer satisfaction should be measured and translated into a number of measurable parameters. Customer satisfaction measurement may be considered as the most reliable feedback system, considering that it provides in an effective, direct, meaningful and objective way the clients’ preferences and expectations. In this way, customer satisfaction is a baseline standard of performance and a possible standard of excellence for any business organisation (Gerson, 1993).

Although, extensive research has defined several alternative approaches for the customer satisfaction evaluation problem, the proposed models and techniques adopt the following main principles:
- The data of the problem are based on the customers’ judgements and should be directly collected from them.

- This is a multivariate evaluation problem given that customer’s global satisfaction depends on a set of variables representing service characteristic dimensions.

- Usually, an additive formula is used in order to aggregate partial evaluations in a global satisfaction measure.

The most important measurement approaches, among others, are (Grigouroudis and Siskos, 2002):

1. *Quantitative methods and data analysis techniques*: descriptive statistics, multiple regression analysis, factor analysis, probit-logit analysis, discriminant analysis, conjoint analysis, and other statistical quantitative methods (DEA, cluster analysis, probability Plotting methods).

2. *Quality approach*: Malcolm Baldridge award, European quality model, ideal point approach, Servqual.


4. *Other methodological approaches*: customer loyalty, Kano’s model, Fornell’s model.

Many of the aforementioned models do not consider the qualitative form of customers’ judgements, although this information is the basic satisfaction input data. Furthermore, in several cases, the measurements are not sufficient enough to analyse in detail customer satisfaction because models’ results are mainly focused on a simple descriptive analysis. The presented multicriteria preference disaggregation method, namely the MUSA (MUlticriteria Satisfaction Analysis) method overcomes these disadvantages. Its principal methodological frame has been developed by Siskos, *et al.* (1998) and Grigouroudis, *et al.* (1999c). The main objectives of the MUSA method are:

1. The evaluation of customers’ satisfaction level, both globally and partially for each of the characteristics of the provided service.

2. The supply of a complete set of results that analyse in depth customers’ preferences and expectations, and explain their satisfaction level.

3. The development of a decision tool with emphasis on the understanding and the applicability of the provided results.

This paper consists of 7 sections. Section 2 is devoted to the development of the multicriteria preference disaggregation MUSA method, while an analytical presentation of the provided results is discussed in section 3. Section 4 presents the reliability evaluation of the method through the assessment of several quantitative stability and fitting indices. A simple illustrative example is given in section 5, while several extensions of the MUSA method are presented in section 6. Finally, section 7 presents some concluding remarks, as well as future research in the context of the proposed method.

### 2. THE MUSA METHOD

#### 2.1 Main principles and notations

The main objective of the proposed MUSA method is the aggregation of individual judgements into a collective value function assuming that client’s global satisfaction depends on a set of $n$ criteria or variables representing service characteristic dimensions. This set of criteria is denoted as
\( \mathbf{X} = (X_1, X_2, \ldots, X_n) \), where a particular criterion \( i \) is represented as a monotonic variable \( X_i \). This way, the evaluation of customer’s satisfaction can be considered as a multicroteria analysis problem.

The required information is collected via a simple questionnaire through which the customers evaluate provided service, i.e. they are asked to express their judgements, namely their global satisfaction and their satisfaction with regard to the set of discrete criteria. A predefined ordinal satisfaction scale is used for these customers’ judgements, as presented in the numerical example of section 5.

The MUSA method assesses global and partial satisfaction functions \( Y^* \) and \( X_i^* \) respectively, given customers’ judgements \( Y \) and \( X_i \). It should be noted that the method follows the principles of ordinal regression analysis under constraints, using linear programming techniques (Jacquet-Lagrèze and Siskos, 1982; Siskos and Yannacopoulos, 1985; Siskos, 1985). The ordinal regression analysis equation has the following form (Table 1 presents model variables):

\[
\begin{align*}
Y^* &= \sum_{i=1}^{n} b_i X_i^* \\
\sum_{i=1}^{n} b_i &= 1
\end{align*}
\]  

(1)

where \( b_i \) is the weight of the \( i \)-th criterion and the value functions \( Y^* \) and \( X_i^* \) are normalised in the interval \([0,100]\), so that \( y_i^* = x_i^{*-1} = 0 \) and \( y_i^u = x_i^{*-u} = 100 \) for \( i = 1, 2, \ldots, n \).

Furthermore, because of the ordinal nature of \( Y \) and \( X_i \) the following preference conditions are assumed:

\[
\begin{align*}
&y_i^m \leq y_i^{m+1} \iff y_i^m \preceq y_i^{m+1} \quad \text{for } m = 1, 2, \ldots, a - 1 \\
x_i^k \leq x_i^{k+1} \iff x_i^k \preceq x_i^{k+1} \quad \text{for } k = 1, 2, \ldots, a_i - 1
\end{align*}
\]

(2)

where \( \preceq \) means “less preferred or indifferent to”.

2.2 Model development

The MUSA method infers an additive collective value function \( Y^* \), and a set of partial satisfaction functions \( X_i^* \) from customers’ judgements. The main objective of the method is to achieve the maximum consistency between the value function \( Y^* \) and the customers’ judgements \( Y \).

Based on the modelling presented in the previous section, and introducing a double-error variable, the ordinal regression equation becomes as follows:

\[
\bar{Y}^* = \sum_{i=1}^{n} b_i X_i^* \bar{Y}^* + \sigma^++ \sigma^-
\]

(3)

where \( \bar{Y}^* \) is the estimation of the global value function \( Y^* \), and \( \sigma^+ \) and \( \sigma^- \) are the overestimation and the underestimation error, respectively.

Equation (3) holds for a customer who has expressed a set of satisfaction judgements. For this reason, a pair of error variables should be assessed for each customer separately (Figure 1).
A careful inspection of equation (3) makes obvious the similarity of the MUSA method with the principles of goal programming modelling, ordinal regression analysis, and particularly with the additive utility models of the UTA family (Jacquet-Lagrèze and Siskos, 1982; Siskos and Yannacopoulos, 1985; Despotis, et al., 1990).

According to the aforementioned definitions and assumptions, the customers’ satisfaction evaluation problem can be formulated as a linear program in which the goal is the minimisation of the sum of errors, under the constraints: (i) ordinal regression equation (3) for each customer, (ii) normalisation constraints for \(Y^*\) and \(X_i^*\) in the interval \([0, 100]\), and (iii) monotonicity constraints for \(Y^*\) and \(X_i^*\).

Removing the monotonicity constraints, the size of the previous LP can be reduced in order to decrease the computational effort required for optimal solution search. This is effected via the introduction of a set of transformation variables, which represent the successive steps of the value functions \(Y^*\) and \(X_i^*\) (Siskos and Yannacopoulos, 1985; Siskos, 1985). The transformation equation can be written as follows:

\[
\begin{align*}
  z_m &= y_m^{n+1} - y_m^* \quad \text{for } m = 1, 2, \ldots, \alpha - 1 \\
  w_{ik} &= b_i x_i^* - b_i x_i^* \quad \text{for } k = 1, 2, \ldots, \alpha - 1 \text{ and } i = 1, 2, \ldots, n
\end{align*}
\]  

(4)

It is very important to mention that using these variables, the linearity of the method is achieved since equation (3) presents a non-linear model (the variables \(Y^*\) and \(X_i^*\), as well as the coefficients \(b_i\) should be estimated).

Using equation (4), the initial variables of the method can be written as:

\[
y_m^* = \sum_{i=1}^{n-1} z_i \quad \text{for } m = 2, 3, \ldots, \alpha, \quad \text{and} \quad b_i x_i^* = \sum_{k=1}^{\alpha - 1} w_{ik} \quad \text{for } k = 2, 3, \ldots, \alpha, \text{ and } i = 1, 2, \ldots, n
\]  

(5)

Therefore, the final form for the LP of the MUSA method can be written:

\[
\begin{align*}
  \text{[min]} \quad F &= \sum_{j=1}^M \sigma_j^* + \sigma_j^- \\
  \text{subject to} \\
  &\sum_{i=1}^{r-1} \sum_{k=1}^{t-1} w_{ik} - \sum_{m=1}^{\alpha - 1} z_m - \sigma_j^* + \sigma_j^- = 0 \quad \text{for } j = 1, 2, \ldots, M \\
  &\sum_{m=1}^{\alpha - 1} z_m = 100 \\
  &\sum_{i=1}^{n} \sum_{k=1}^{\alpha - 1} w_{ik} = 100 \\
  &z_m, w_{ik}, \sigma_j^*, \sigma_j^- \geq 0 \quad \forall \; m, i, j, k
\end{align*}
\]  

(6)
where \( t_j \) and \( t_{ji} \) are the judgements of the \( j \)-th customer for global and partial satisfaction with \( y'_j \in Y = \{ y_1', y_2', \ldots, y_i', \ldots, y_n' \} \) and \( x_j' \in X_j = \{ x_1^j, x_2^j, \ldots, x_i^j, \ldots, x_n^j \} \) for \( i = 1, 2, \ldots, n \), and \( M \) is the number of customers.

The previous LP has \( M + 2 \) constraints and \( 2M + (a - 1) + \sum_{i=1}^{n} (\alpha_i - 1) \) variables, so the optimal solution has many degrees of freedom. For this reason, a stability analysis is considered necessary, as presented in the next section. Furthermore, it should be noted that following Wagner’s modelling for quantitative regression analysis using linear programming techniques, the dual LP may be solved in order to reduce the complexity of obtaining an optimal solution (Wagner, 1959).

The calculation of the initial model variables is based on the solution of the previous LP, since:

\[
\begin{align*}
 b_i &= \frac{\sum_{i=1}^{n} w_{ij}}{100} \text{ for } i = 1, 2, \ldots, n \\
 y_m &= \sum_{i=1}^{n} z_{ij} \text{ and } x_i^k &= \frac{100 \sum_{j=1}^{n} w_{ij}}{\sum_{j=1}^{n} w_{ij}} \text{ for } m = 2, 3, \ldots, \alpha, \ i = 1, 2, \ldots, n \text{ and } k = 2, 3, \ldots, \alpha_i
\end{align*}
\]

2.3 Stability analysis

The stability analysis is considered as a post-optimality analysis problem, considering that the MUSA method is based on a linear programming modelling. In this context, it should be noted that in several cases, particularly in large-scale LPs, the problem of multiple or near optimal solutions appears.

The MUSA method applies a heuristic method for near optimal solutions search (Siskos, 1984). These solutions have some desired properties, while the heuristic technique is based on the following:

- In several cases, the optimal solutions are not the most interesting, given the uncertainty of the model parameters and the preferences of the decision-maker (Van de Panne, 1975).
- The number of the optimal or near optimal solutions is often huge. Therefore an exhaustive search method (reverse simplex, Manas-Nedoma algorithms) requires a lot of computational effort.

The post-optimal solutions space is defined by the polyhedron \( \{ F \leq F^* + \varepsilon \}, \) all the constraints of LP(6), where \( F^* \) is the optimal value for the objective function of LP (6), and \( \varepsilon \) is a small percentage of \( F^* \). According to the aforementioned remarks, during the post optimality analysis stage of the MUSA method, \( n \) linear programs (equal to the number of criteria) are formulated and solved. Each linear program maximises the weight of a criterion and has the following form:

\[
\begin{align*}
 [\text{max}] F' &= \sum_{k=1}^{n} w_{jk} \text{ for } i = 1, 2, \ldots, n \\
 \text{subject to} \\
 F' &\leq F^* + \varepsilon \\
 \text{all the constraints of LP (6)}
\end{align*}
\]

The average of the optimal solutions given by the \( n \) LPs (8) may be considered as the final solution of the problem. In case of instability, a large variation of the provided solutions appears, and this average solution is less representative.
3. RESULTS OF THE MUSA METHOD

3.1 Value functions and criteria weights

The estimated value functions are the most important results of the MUSA method, considering that they show the real value, in a normalised interval [0, 100], that customers give for each level of the global or partial ordinal satisfaction scale. The form of these functions indicates the customers’ degree of demanding. Figure 2 presents an example of 3 (global or partial) value functions referring to customer groups with different demanding levels:

- **Neutral customers**: the value function has a linear form; the more satisfied these customers express they are, the higher the percentage of their fulfilled expectations is.

- **Demanding customers**: this refers to the case of a convex value function; customers are not really satisfied, unless they receive the best quality level.

- **Non-demanding customers**: this refers to the case of a concave value function; customers express that they are satisfied, although only a small portion of their expectations is fulfilled.

The customers’ satisfaction global and partial value functions \( Y^* \) and \( X_i^* \), respectively, are mentioned as additive and marginal value or utility functions, and their properties are determined in the context of multicriteria analysis. The collective value function \( Y^* \), particularly, represents the customers’ preference value system, and indicates the consequences of the satisfaction criteria. Moreover, the MUSA method assumes that \( Y^* \) and \( X_i^* \) are monotonic, non-decreasing, discrete (piecewise linear) functions.

The satisfaction criteria weights represent the relative importance of the assessed satisfaction dimensions, given that \( \sum_{i=1}^{n} b_i = 1 \). Thus, the decision of whether the customers consider a satisfaction dimension important is also based on the number of the assessed criteria (see section 3.5). The properties of the weights are also determined in the context of multicriteria analysis, and it should be noted that the weights are basically value trade-offs among the criteria.

3.2 Average satisfaction indices

The assessment of a performance norm, globally and per satisfaction criterion as well, may be very useful in customer satisfaction analysis and benchmarking. The average global and partial satisfaction indices, \( S \) and \( S_i \), respectively, are used for this purpose, and can be assessed according to the following formulas:

\[
\begin{align*}
S &= \frac{1}{100} \sum_{n=1}^{m} p^n_\alpha y^n_\alpha \\
S_i &= \frac{1}{100} \sum_{i=1}^{n} p^i_{\alpha} x^i_{\alpha} & \text{for } i = 1, 2, \ldots, n
\end{align*}
\]

(9)

where \( p^n_\alpha \) and \( p^i_{\alpha} \) are the frequencies of customers belonging to the \( y^n_\alpha \) and \( x^i_{\alpha} \) satisfaction levels, respectively.
It should be noted that the average satisfaction indices are basically the mean value of the global or partial value functions and they are normalised in the interval \([0, 100\%]\).

### 3.3 Average demanding indices

The need to assess a set of average demanding indices has been raised in section 3.1, given the following advantages:

- A quantitative measure is assessed for the concept of customers’ demanding.
- The information provided by the added values is fully exploited.

The average global and partial demanding indices, \(D\) and \(D_i\), respectively, are assessed as follows:

\[
D = \frac{\sum_{m=1}^{a-1} \left( \frac{100(m-1)}{\alpha - 1} - y^*m \right)}{100 \sum_{m=1}^{a-1} \frac{m-1}{\alpha - 1}} = 1 - \frac{\bar{y}^*}{50} \quad \text{for} \quad \alpha > 2
\]

\[
D_i = \frac{\sum_{k=1}^{a-i} \left( \frac{100(k-1)}{\alpha_i - 1} - x^*_i \right)}{100 \sum_{k=1}^{a-i} \frac{k-1}{\alpha_i - 1}} = 1 - \frac{\bar{x}^*_i}{50} \quad \text{for} \quad \alpha_i > 2 \quad \text{and} \quad i = 1, 2, \ldots, n
\]

where \(\bar{y}^*\) and \(\bar{x}^*_i\) are the mean values of functions \(Y^*\) and \(X^*_i\) respectively.

The average demanding indices are normalised in the interval \([-1, 1]\) and the following possible cases hold (see also section 3.1):

- \(D = 1\) or \(D_i = 1\): customers have the maximum demanding level.
- \(D = 0\) or \(D_i = 0\): this case refers to the neutral customers.
- \(D = -1\) or \(D_i = -1\): customers have the minimum demanding level.

These indices represent the average deviation of the estimated value curves from a “normal” (linear) function. This means that the demanding indices can take different values for different levels of the ordinal satisfaction scale. For example, a sigmoid value function can give a zero average demanding index. In this case, if additional detailed analysis is required, the \(z_n\) and \(w_d/h_i\) variables, as denoted in (4), can be considered as a set of discrete demanding functions.

Demanding indices can be used for customer behaviour analysis, and they can also indicate the extent of company’s improvement efforts: the higher the value of the demanding index, the more the satisfaction level should be improved in order to fulfil customers’ expectations.

### 3.4 Average improvement indices

The output of improvement efforts depends on the importance of the satisfaction dimensions and their contribution to dissatisfaction as well. The average improvement indices can show the improvement margins on a specific criterion, and they are assessed according to the following equation:

\[
I_i = b_i (1 - S_i) \quad \text{for} \quad i = 1, 2, \ldots, n
\]
These indices are normalised in the interval \([0, 1]\) and it can be proved that \(I_i = 1 \Leftrightarrow b_i = 1 \land S_i = 0\) and \(I_i = 0 \Leftrightarrow b_i = 0 \lor S_i = 1\) (for \(i = 1, 2, \ldots, n\)). The average improvement indices are used for the development of a series of improvement diagrams, as presented in section 3.6.

### 3.5 Action diagrams

Combining weights and average satisfaction indices, a series of action diagrams can be developed. These diagrams indicate the strong and the weak points of customer satisfaction, and define the required improvement efforts.

These diagrams are also mentioned as decision, strategic, perceptual, and performance-importance maps (Dutka, 1995; Customers Satisfaction Council, 1995; Naumann and Giel, 1995), or gap analysis (Hill, 1996; Woodruff and Gardial, 1996; Vavra, 1997), and they are similar to SWOT (Strengths-Weaknesses-Opportunities-Threats) analysis.

Each of these maps is divided into quadrants, according to performance (high/low) and importance (high/low) that may be used to classify actions (Figure 3):

- **Status quo** (low performance and low importance): Generally, no action is required.
- **Leverage opportunity** (high performance/high importance): These areas can be used as advantage against competition.
- **Transfer resources** (high performance/low importance): Company’s resources may be better used elsewhere.
- **Action opportunity** (low performance/high importance): These are the criteria that need attention.

This grid can be used in order to identify priorities for improvement. The bottom right quadrant is obviously the first priority, for the attributes are important to customers but company’s performance is rated moderately low. The second priority may be given to the satisfaction criteria in the top right quadrant, especially if there is room for improvement. The third priority issues are indicated in the bottom left quadrant; although these issues are not terribly pertinent at the time of the analysis, they may be more important in the future, and company’s performance is certainly not good. Finally, last priority for improvement should be given to the criteria in the top left quadrant because this category is the least important and company’s performance is relatively good. Apparently, priorities for improvement may vary among different companies, depending on the potential capabilities of improving the particular category.

For the development of these diagrams, two alternative approaches can be used:

1. **Raw action diagrams**: They use the weights and the average satisfaction indices as they are calculated by the MUSA method. The importance axis refers to the criteria weights \(b_i\), which take values in the range \([0, 1]\), and it is assumed that a criterion is important if \(b_i > 1/n\), considering that the weights are based on the number of criteria. On the other hand, the performance axis refers to the average satisfaction indices \(S_i\), which are also normalised in the interval \([0, 1]\). The cut-off level defines if a particular criterion has high or low performance,
and it has been chosen to be equal to 0.5 (50%). This is a rather arbitrary assumption, which can be reconsidered depending upon the case.

2. Relative action diagrams: These diagrams use the relative variables $b'_i$ and $S'_i$ in order to overcome the assessment problem of the cut-off level for the importance and the performance axis. This way, the cut-off level for axes is recalculated as the centroid of all points in the diagram (Grigoroudis and Siskos, 2002). This type of diagram is very useful if points are concentrated in a small area because of the low-variation appearing for the average satisfaction indices (e.g. case of a high competitive market).

3.6 Improvement diagrams

The action diagrams can indicate which satisfaction dimensions should be improved, but they cannot determine the output or the extent of the improvement efforts. For this reason, combining the average improvement and demanding indices, a series of improvement diagrams can be developed.

As shown in Figure 4, each of these maps is divided into quadrants according to demanding (high/low) and effectiveness (high/low) that may be used to rank improvement priorities:

- **1st priority**: this area indicates direct improvement actions since these dimensions are highly effective and customers are not demanding.
- **2nd priority**: it includes satisfaction dimensions that have either a low demanding index or a high improvement index.
- **3rd priority**: it refers to satisfaction dimensions that have a small improvement margin and need substantial effort.

![Insert Figure 4](image)

Similar to the previous section, there are 2 types of improvement diagrams:

1. **Raw improvement diagrams**: They use the average improvement and demanding indices as they are calculated by the MUSA method.

2. **Relative improvement diagrams**: The cut-off level for axes is recalculated as the centroid of all points in the diagram.

4. RELIABILITY EVALUATION

The reliability evaluation of the results is mainly related to the following points:

- the fitting level to the customer satisfaction data, and
- the stability of the post-optimality analysis results.

Some quantitative measures for the evaluation of the results provided by the MUSA method are presented in the following sections (4.1-4.3).

4.1 Average fitting index

The fitting level of the MUSA method refers to the assessment of a preference collective value system (value functions, weights, etc.) for the set of customers with the minimum possible errors.
For this reason, the optimal values of the error variables indicate the reliability of the value system that is evaluated.

The Average Fitting Index (AFI) depends on the optimum error level and the number of customers:

\[ AFI = 1 - \frac{F^*}{100 \cdot M} \]  

(12)

The AFI is normalised in the interval [0, 1], and it is equal to 1 if \( F^* = 0 \), i.e. when the method is able of evaluating a preference value system with zero errors. Similarly, the AFI takes the value 0 only when the pairs of the error variables \( \sigma_j^+ \) and \( \sigma_j^- \) take the maximum possible values. It is easy to prove that \( \sigma_j^+ \cdot \sigma_j^- = 0 \ \forall j \), i.e. the optimal solution has at least one zero error variable for each customer, given that the MUSA method is similar to goal programming modelling (Charnes and Cooper, 1977).

4.2 Other fitting indicators

The variance diagram of the additive value curve depends upon the estimated satisfaction values and the optimal values of the error variables as well. For developing this diagram, the maximum and the minimum satisfaction curves \( y_{\text{max}}^* \) and \( y_{\text{min}}^* \) respectively, are calculated for each level \( m \) of the ordinal satisfaction scale, using the following formula:

\[
\begin{cases}
  y_{\text{max}}^m = \max_j \{ \tilde{y}_j^m \} \\
  y_{\text{min}}^m = \min_j \{ \tilde{y}_j^m \}
\end{cases} \quad \text{for } m = 1, 2, \ldots, \alpha
\]  

(13)

where \( \tilde{y}_j^m \) is the evaluated satisfaction value for customer \( j \), with \( \tilde{y}_j^m = y_j^m + \sigma_j^+ - \sigma_j^- \).

This variance diagram shows the value range that the customers’ set gives for each level of the ordinal satisfaction scale. Thus, it can be considered as a confidence interval for the estimated additive value function.

The prediction table of global satisfaction is developed in a similar way according to the next steps:

1. For each customer \( j \), the evaluated satisfaction value \( \tilde{y}_j^m \) is calculated.

2. Based on the previous value, for each customer \( j \), the evaluated satisfaction level \( \tilde{y}_j^m \) is calculated according to the formula:

\[
\tilde{y}_j^m = \begin{cases}
  y_j^1 & \text{if } \frac{y_j^m}{2} \leq \frac{Y^2}{2} \\
  y_j^2 & \text{if } \frac{y_j^m}{2} < \frac{y_j^m}{2} \leq \frac{y_j^m}{2} + y_j^2 \\
  \vdots & \\
  y_j^\alpha & \text{if } \frac{y_j^m}{2} > \frac{100 + y_{\text{min}}}{2}
\end{cases}
\]  

(14)

3. Using the actual (as expressed by the customers) and the estimated level of global satisfaction, \( y_j^m \) and \( \tilde{y}_j^m \) accordingly, the number of customers belonging to each of these levels is calculated.
4. The general form of a prediction table is presented in Figure 5, and includes the following results for each actual and evaluated satisfaction level:

- \( N_{m_1, m_2} \): the number of customers that have declared to belong to global satisfaction level \( m_1 \), while the model classifies them to level \( m_2 \).

- \( R_{m_1, m_2} \): the percentage of customers of actual global satisfaction level \( m_1 \), that the model classifies to level \( m_2 \), with \( R_{m_1, m_2} = N_{m_1, m_2} / \sum_{m_1=1}^{\alpha} N_{m_1, m_2}, \forall m_1, m_2 \).

- \( C_{m_1, m_2} \): the percentage of customers of estimated global satisfaction level \( m_1 \), that have declared to belong to level \( m_2 \), with \( C_{m_1, m_2} = N_{m_1, m_2} / \sum_{m_1=1}^{\alpha} N_{m_1, m_2}, \forall m_1, m_2 \).

The Overall Prediction Level (\( OPL \)) is based on the sum of the main diagonal cells of the prediction table, and it represents the percentage of correctly classified customers:

\[
OPL = \frac{\sum_{m_1=1}^{\alpha} N_{m_1, m_1}}{\sum_{m_1=1}^{\alpha} \sum_{m_2=1}^{\alpha} N_{m_1, m_2}}
\]  

(15)

In general, it should be mentioned that the fitness of the MUSA method is not satisfactory when a high percentage of customers appears away from the main diagonal of the prediction table, i.e. a significant number of customers having declared to be very satisfied is predicted to have a low satisfaction level and vice versa.

4.3 Average stability index

The stability of the results provided by the post-optimality analysis is not related to the degree of fitness of the MUSA method. More specifically, during the post-optimality stage, \( n \) LPs are formulated and solved, which maximise repeatedly the weight of each criterion. The mean value of the weights of these LPs is taken as the final solution, and the observed variance in the post-optimality matrix indicates the degree of instability of the results. Thus, an Average Stability Index (\( ASI \)) may be assessed as the mean value of the normalised standard deviation of the estimated weights:

\[
ASI = 1 - \frac{1}{n} \sum_{i=1}^{n} \sqrt{n \sum_{j=1}^{\alpha} \left( b_{ij}^j \right)^2 - \left( \sum_{j=1}^{\alpha} b_{ij}^j \right)^2} \quad \text{for} \ i \neq j \quad \forall i, j
\]  

(16)

where \( b_{ij}^j \) is the estimated weight of the criterion \( i \) in the \( j \)-th post-optimality analysis LP.

The \( AFI \) is normalised in the interval \([0, 1]\), and it should be noted that when this index takes its maximum value, \( ASI = 1 \iff b_{ij}^j = b_{ij} \ \forall i, j \). On the other hand, if the \( ASI \) takes its minimum value, \( ASI = 0 \iff b_{ij}^j = 100 \ (\text{for} \ i = j) \land b_{ij}^j = 0 \ (\text{for} \ i \neq j) \ \forall i, j \).

Generally, apart the \( ASI \), the variance table of the weights (see section 6) is also able to provide valuable information for the stability analysis of the results provided by the MUSA method. This table can give a confidence interval for the estimated weights, and can identify possible
competitiveness in the criteria set, i.e. the existence of certain customer groups with different importance levels for the satisfaction criteria.

5. A NUMERICAL EXAMPLE

Assume a case of a customer satisfaction survey conducted for a service providing business organisation, with the following data:

1. Customers’ global satisfaction depends on 3 main criteria: product, purchase process, and additional service. Although this is not a consistent family of criteria, it is used for illustrating the implementation of the MUSA method.

2. A 3-point ordinal satisfaction scale is used, which is the same for both the global and the partial satisfaction judgements.

3. Table 2 presents the data of the satisfaction survey that includes satisfaction judgements for a set of 20 customers.

During the 1st implementation stage of the MUSA method, the initial LP (6) is formulated and solved using the data of Table 2. As shown in Table 3, the sum of errors equals 0 in the optimal solution found. It is important to mention the existence of multiple optimal solutions in this initial LP.

The 2nd implementation stage of the method concerns the post-optimality analysis, where 3 LPs (equal to the number of problem criteria) are solved, each of them maximising the weight of the corresponding satisfaction criterion. The final solution of the model variables is calculated as the average of the post-optimal solutions obtained from the aforementioned LPs (Table 4). Despite the small sample size, it is worth to mention the relatively high stability of the results.

In the last implementation stage, the main results of the MUSA method (criteria weights, average satisfaction, demanding, and improvement indices) are calculated, as shown in Table 5.
Moreover, given the formula (7) and the information provided by Tables 4-5, the value function curves (Figure 6) and the action and improvement diagrams (Figures 7-8) are created.

![Insert Figure 6]

![Insert Figure 7]

![Insert Figure 8]

Given the results of the numerical example, the following points raise:

1. A very low satisfaction level appears for the customers’ set (average global satisfaction index 50.33%). This result is also justified by the statistical frequencies of the sample (30% of the customers are globally dissatisfied).

2. Regarding the satisfaction dimensions, the criterion of the “purchase process” seems to be the most important (weight 47.17%), while at the same time it presents the lowest satisfaction index (46.74%). This result is also demonstrated in the related action diagram that suggests this particular criterion as a critical satisfaction dimension.

Therefore, the improvement efforts should focus on the “purchase process” criterion as the related improvement diagram also suggests. Additionally, other improvement efforts may concern the criterion of “additional service” mostly due to the observed low demanding level.

The reliability evaluation analysis shows the high level of fitting and stability of the provided final results. For example, the average fitting index is calculated as \( AFI = 100\% \), since \( F^* = 0 \). Also, the low variation observed during the post-optimality analysis stage (Table 4) is reflected in the high value of the average stability index: \( ASI = 91.77\% \). More analytically, Table 6 presents the variation observed for the most important variables of the method, during the post-optimality analysis stage. The maximum and minimum values appearing may be considered as a confidence interval for the estimation of these variables. Furthermore, using the variation of the estimated added value curve \( \tilde{Y}^* \), a similar confidence interval for the average global satisfaction index can be calculated. In this way, estimated \( S \) varies from 49.0% to 52.0% during post-optimality analysis. Finally, the overall prediction level demonstrates the 100% fitness of the model (\( OPL = 100\% \)), since all the non-zero elements belong to the main diagonal of the prediction table of global satisfaction.

![Insert Table 6]

6. EXTENSIONS OF THE MUSA METHOD

6.1 Potential implementation problems
The potential implementation problems of the MUSA method concern the model assumptions and the quality of the collected data, which is however something common in all the regression analysis models.

The logical inconsistency of customer satisfaction data affects directly the reliability and the stability of the results. Examples of such inconsistencies are presented when a customer answers that she/he is “Very satisfied” according to the total set of criteria, while at the same time declares that she/he is globally “Dissatisfied, and vice versa. The main reasons of this problem are:

1. The satisfaction criteria set is not a consistent family of criteria.
2. The customers are not rational decision-makers.

During the implementation process of the MUSA method, a preliminary stage for searching such inconsistencies should be applied. If the problem appears in a small portion of customers, the particular data should be removed, while in the opposite case the defined satisfaction criteria set should be reconsidered.

Another problem that may appear concerns the existence of distinguished customer groups with different preference value systems (value functions, criteria weights, etc.). This problem can be noticed by the high variance of the variables during the post-optimality analysis and is due to the collective nature of the MUSA method. The segmentation of the total set of customers into smaller groups according to particular characteristics (e.g. age, sex) is the most reliable solution to the previous problem.

Other potential problems that affect the reliability of the provided results include the following:

- In several cases with unstable results, it appears that $y^{*m} = y^{*m+1}$ or $x_{i}^{*k} = x_{i}^{*k+1}$ (Jacquet-Lagrèze and Siskos, 1982).
- Cases where $b_j = 0$ for some criteria $X_j$ should be avoided.

Assessing a set of preference threshold may overcome these problems, as discussed in the next section.

6.2 Strictly increasing value functions

In this extension, it is assumed that $Y^*$ and $X_i^*$ are monotonic and strictly increasing functions in order to consider the strict preferential order of the scales of some or all the satisfaction criteria. Taking into account the hypothesis of strict preferences, the conditions of equation (2) become as follows:

$$
\begin{align*}
    y^{*m} < y^{*m+1} & \iff y^m < y^{m+1} \quad \text{for } m = 1, 2, \ldots, a \\
x_i^{*k} < x_i^{*k+1} & \iff x_i^k < x_i^{k+1} \quad \text{for } k = 1, 2, \ldots, a_i - 1 \text{ and } i = 1, 2, \ldots, n
\end{align*}
$$

(17)

where $< \text{ means “strictly less preferred”}$.

Based on (17) and the definition of $z_m$ and $w_{ik}$ variables, the following conditions occur:

$$
\begin{align*}
    y^{*m+1} - y^m & \geq \gamma \iff z_m \geq \gamma \iff z'_m \geq 0 \quad \text{for } m = 1, 2, \ldots, a \\
x_i^{*k+1} - x_i^{*k} & \geq \gamma_i \iff w_{ik} \geq \gamma_i \iff w'_{ik} \geq 0 \quad \text{for } k = 1, 2, \ldots, a_i - 1 \text{ and } i = 1, 2, \ldots, n
\end{align*}
$$

(18)

where $\gamma$ and $\gamma_i$ are the preference thresholds for the value functions $Y^*$ and $X_i^*$, respectively, with $\gamma, \gamma_i > 0$, and it is set that $z_m = z'_m + \gamma$ and $w_{ik} = w'_{ik} + \gamma_i$. 

The thresholds $\gamma$ and $\gamma_i$ represent the minimum step of increase for functions $Y^*$ and $X_i^*$, respectively (Figure 9), and it can be proved that in this case the minimum weight of a criterion $X_i$ becomes $\gamma_i (\alpha_i - 1)$.

Using the previous formulas, the generalised MUSA method reads:

$$\min \{ F = \sum_{j=1}^{M} \sigma_j^+ + \sigma_j^- \}$$

subject to

$$\sum_{i=1}^{t} \sum_{k=1}^{l} w_{i,k} - \sum_{m=1}^{n} z_m^* - \sigma_j^* + \sigma_j^- = \gamma_j (t_j - 1) - \sum_{i=1}^{M} \gamma_i (t_{ij} - 1) \quad \text{for } j = 1, 2, \ldots, M$$

$$\sum_{m=1}^{n} z_m^* = 100 - \gamma (\alpha - 1)$$

$$\sum_{i=1}^{t} \sum_{k=1}^{l} w_{i,k} = 100 - \sum_{i=1}^{M} \gamma_i (\alpha_i - 1)$$

$$\gamma_j, \gamma_i, z_m^*, w_{i,k}, \sigma_j^*, \sigma_j^- \geq 0 \quad \forall \, m, i, j, k$$

The proposed extension consists the generalised form of the MUSA method, since the basic form of section 2.2 is a special case where $\gamma = \gamma_i = 0, \forall i$.

The post-optimality analysis results of this extended version for the numerical example of section 5 are presented in Table 7, while Table 8 shows a comparison analysis between the basic and the generalised MUSA method. In this table, the significant improvement of the achieved stability should be noted: the mean standard deviation of the results obtained during the post-optimality analysis stage decreases from 3.9 to 0.8.

It should be emphasised that $\gamma$ and $\gamma_i$ variables should be chosen in such a way that $\gamma (\alpha - 1) \leq 100$ and $\sum_{i=1}^{n} \gamma_i (\alpha_i - 1) \leq 100$, in order to avoid negative values in the right-hand constraints of LP (19). For example, in case where $\gamma = \gamma_i, \forall i$, the previous constraints take the following form:

$$\gamma \leq \min \left\{ \frac{100}{(\alpha - 1)}, \frac{100}{\sum_{i=1}^{n} (\alpha_i - 1)} \right\}$$

(20)
Usually, the preference thresholds are selected as small numbers in the interval \([0, 100]\) (e.g. \(\gamma_{ij}^* = 2\) as shown in Table 8), while future research may focus on the development of an analytical procedure for the selection of appropriate values for these variables.

### 6.3 Multiple criteria levels

In several cases, during the assessment process of a consistent family of criteria, it seems rather useful to assume a value or treelike structure. This structure is also mentioned as “value tree” or “value hierarchy” (Keeney and Raiffa, 1976; Keeney, 1992; Kirkwood, 1997).

The first criteria level includes the main satisfaction dimensions in a general form (e.g. personnel), while the second level considers more detailed characteristics (personnel’s friendliness, skills, etc.). It should be noted that the number of levels might not be uniform across a value hierarchy. The total set of main criteria, as well as each of the subcriteria sets should satisfy the properties of a consistent family of criteria.

In this particular case, the formulation of the MUSA method is based on the additional assessed variables of Table 9, and therefore the additional ordinal regression analysis equations read:

\[
\begin{align*}
X_i^* &= \sum_{q=1}^{\gamma_i} b_{iq} X_{iq}^* \\
\sum_{q=1}^{\gamma_i} b_{iq} &= 1
\end{align*}
\]  

(21)

where the value functions \(X_{iq}^*\) are normalised in the interval \([0, 100]\).

Similarly to (4), the new additional variables representing the successive steps of the value functions \(X_{iq}^*\) with \(w_{ijk} = b_{ij} x_{iq}^{*k+1} - b_{ij} x_{iq}^{*k}\) for \(k = 1, 2, ..., \alpha_{ij} - 1\), \(q = 1, 2, ..., n_i\) and \(i = 1, 2, ..., n\).

Thus, the LP (6) takes the following form:

\[
\begin{align*}
\left[ \min \right] F &= \sum_{j=1}^{M} \sigma_j^* + \sigma_j^- + \sum_{j=1}^{M} \sum_{i=1}^{n} \sigma_{ji}^* + \sigma_{ji}^- \\
\text{subject to} \quad &\sum_{q=1}^{\gamma_i} \sum_{k=1}^{\alpha_{ij} - 1} w_{ijk} - \sum_{k=1}^{\alpha_{ij} - 1} w_{ijk} - \sigma_{ji}^* + \sigma_{ji}^- = 0 \quad \text{for } i = 1, 2, ..., n \text{ and } j = 1, 2, ..., M \\
&\sum_{j=1}^{n} \sum_{q=1}^{\gamma_i} \sum_{k=1}^{\alpha_{ij} - 1} w_{ijk} = 100 \\
&\text{all the constraints of LP(6)} \\
&w_{ijk}, \sigma_{ji}^*, \sigma_{ji}^- \geq 0 \quad \forall i, j, k, q
\end{align*}
\]  

(22)

The new model variables are calculated according to the following formulas:
The stability analysis of LP (22) can also be considered as a post-optimality problem, where \( \sum n_i \) LPs are formulated and solved, each of them maximising the weight \( b_y \) of every subcriterion. The LPs have the following form:

\[
\begin{align*}
\left\{ \begin{array}{l}
\text{max} \quad F' = \sum_{k=1}^{n_i - 1} w_{ik} \\
\text{subject to} \quad F \leq F^* + \varepsilon \\
\text{all the constraints of LP(22)}
\end{array} \right.
\]

where \( F^* \) is the optimal value of the objective function of LP(22), and \( \varepsilon \) is a small percentage of \( F^* \).

### 6.4 Alternative objective functions

This section discusses the issue of selecting alternative objective functions during the post-optimality analysis process. The basic form of the MUSA method (section 2) proposes the solution of \( n \) LPs that maximise the weight \( b_i \) of each criterion.

Alternatively, the simultaneous solution of \( n \) LPs that minimise the weight \( b_i \) of each criterion may be considered. In this case (MUSA I model), \( 2n \) LPs should be formulated and solved:

\[
\begin{align*}
\left\{ \begin{array}{l}
\text{max} \quad F' = \sum_{k=1}^{n_i - 1} w_{ik} \\
\text{min} \quad F' = \sum_{k=1}^{n_i - 1} w_{ik} \quad \forall i \\
\text{subject to} \quad \text{all the constraints of LP(22)}
\end{array} \right.
\]

Another alternative approach considers the search of near optimal solutions that maximise the preference thresholds. This approach overcomes the problem of selecting appropriate values for these parameters that are known to affect the stability of the model (Srinivasan and Shocker, 1973), while the estimated optimal values \( \gamma \) and \( \gamma_i \) maximise the discrimination of the preference conditions (Beuthe and Scannella, 2001). In this particular case (MUSA II model), the post-optimality analysis includes the solution of \( n+1 \) LPs of the following form:

\[
\begin{align*}
\left\{ \begin{array}{l}
\text{max} \quad F' = \gamma \\
\text{max} \quad F' = \gamma_i \quad \forall i \\
\text{subject to} \quad F \leq F^* + \varepsilon \\
\text{all the constraints of LP(19)}
\end{array} \right.
\]

Similarly to the previous case, the maximisation of the successive steps of the value functions \( Y^* \) and \( X_i^* \) (\( w_{ik} \) and \( z_{ik} \) variables) may be considered during the post-optimality process (Beuthe and Scannella, 1996). In this particular approach (MUSA III model), the stability analysis includes the solution of \( (\alpha - 1) + \sum_{i=1}^{n} (\alpha_i - 1) \) LPs of the following form:
\[ \begin{align*}
\max F' &= z_m \quad \text{and} \quad \max F' &= w_{ik} \quad \forall m,i,k \\
\text{subject to} & \quad \text{all the constraints of LP(8)}
\end{align*} \tag{27} \]

A final alternative approach, in the context of ordinal regression analysis, considers the minimisation of the difference between the maximum and the minimum value of the error variables \( \sigma_j^+ \) and \( \sigma_j^- \) in the case where \( F' > 0 \) (Despotis, et al., 1990). This particular approach (MUSA IV model) corresponds to the minimisation of the \( L_\infty \) norm of errors. So, if \( m_e \) is the maximum value of the error variables \( \sigma_j^+ \) and \( \sigma_j^- \), the LP of the post-optimality analysis takes the following form:

\[ \begin{align*}
\min F' &= m_e \\
\text{subject to} & \quad m_e - \sigma_j^+ \geq 0 \quad \forall j \\
& \quad m_e - \sigma_j^- \geq \gamma_j \quad \forall j \\
& \quad \text{all the constraints of LP(8)}
\end{align*} \tag{28} \]

Discussing the nature of the proposed extensions, the following points raise:

1. The satisfaction criteria are usually competitive and therefore it is not necessary to consider simultaneously the maximisation and minimisation of the criteria weights. For this reason, the MUSA I method is very similar to the generalised MUSA method (for \( \gamma_i = 0 \)).

2. The MUSA III version is an extension of MUSA II, since \( \gamma_i \leq \max \{z_m\} \) and \( \gamma_i \leq \min \{w_{ik}\} \quad \forall i \).

3. Usually, the minimisation of the \( L_\infty \) norm of errors (MUSA IV) is of limited support to this particular case, although it is an important tool of the post-optimality problem in the context of ordinal regression analysis. This extension distributes equally the error values to the total set of customers. This way, the collective method is not able to “correct” potential discriminated judgements. Additionally, it should be noted that this version applies only if \( F' > 0 \), and therefore, it cannot overcome the stability analysis problem (multiple or near optimal solutions).

The implementation of all different approaches to the numerical example of section 5 gives similar results, as presented in Table 10, and for this reason it reveals the high stability of the results for this particular data set.

7. CONCLUDING REMARKS AND FUTURE RESEARCH

The proposed MUSA method is based on the principles of multicriteria analysis, and particularly on aggregation-dissaggregation approach and linear programming modelling. The implementation of the method in customer satisfaction surveys is able to evaluate quantitative global and partial satisfaction levels and to determine the strong and the weak points of a business organisation.

The main advantage of the MUSA method is that it fully considers the qualitative form of customers’ judgements and preferences, as they are expressed in a customer satisfaction survey.
The MUSA method avoids the arbitrary quantification of the collected information, because, as emphasised in the paper, the coding of the qualitative scale is a result, not an input to the proposed methodology. This does not occur in a simple linear regression analysis. Other advantages of the method include the following:

1. The post-optimality analysis stage gives the ability to achieve a sufficient stability level concerning the provided results, while the linear programming formulation offers a flexible model development.

2. The provided results are focused not only on the descriptive analysis of customer satisfaction data but they are also able to assess an integrated benchmarking system. This way they offer a complete information set and they include: value functions, criteria weights, average satisfaction, demanding, and improvement indices, action and improvement diagrams, average stability and fitting indices, prediction table of global satisfaction, etc. These results are sufficient enough to analyse in detail the satisfaction evaluation problem, and to assess the reliability of the method’s implementation.

3. A significant effort has been devoted in order for all the provided results to be easily and directly understood. For this reason, the indices’ sets are assessed in a normalised $[0, 100\%]$ interval.

The implementation of the MUSA method in satisfaction evaluation problems refers mostly to customers or employees of business organisations. Siskos, et al. (1998), Mihelis, et al. (2001), and Grigoroudis, et al. (1999b), present several real world applications of this type. Furthermore, satisfaction evaluation problems may refer to any human activity or social field like evaluation of educational systems (Siskos, et al., 2001), voters’ satisfaction, shareholders’ or investors’ satisfaction, expectations and needs of a labour market, or benchmarking for business organisations.

Future research regarding the MUSA method is mainly focused on comparison analysis with other alternative satisfaction measurement approaches like statistical models, data analysis techniques, fuzzy sets, and other advanced prediction methods (e.g. neural networks). Moreover, the problem of selecting appropriate values for the parameters of the method (preference thresholds, $\varepsilon$ value) and its impact to the reliability and stability of the provided results should be studied.

The implementation of the MUSA method requires completely and correctly answered questionnaires as input data, which cannot always be achieved. Missing data analysis and data mining techniques may be used to overcome this problem by filling in the empty cells in the data table (Matsatsinis, et al., 2001).

Other possible extensions of the method include:

- The development of an extended MUSA method in a customer satisfaction survey for a set of competitive companies, given that the currently presented version is focused on the satisfaction evaluation problem for a single business organisation.

- The assessment of a “critical” satisfaction level, that can relate customer satisfaction level and repurchase probability. Hill (1996) notes several research efforts for the determination of a customer tolerance band. Furthermore, combining MUSA method with several brand choice models, the segmentation of the total set of customers into smaller groups with different loyalty levels can be achieved. A pilot survey in the context of multicriteria analysis is proposed by Grigoroudis, et al. (1999a).

Finally, it is interesting to analyse the relation between the results of the MUSA method and the financial indices (market share, profit, etc.) of a business organisation. Although customer
satisfaction is a necessary but not a sufficient condition for the financial viability, several researches have shown that there is a significant correlation among satisfaction level, customer loyalty, and company’s profit (Dutka, 1995; Naumann and Giel, 1995).
REFERENCES


Table 1: Variables of the MUSA method

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>client’s global satisfaction</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>number of global satisfaction levels</td>
</tr>
<tr>
<td>$y_m^m$</td>
<td>the $m$-th global satisfaction level ($m=1,2,\ldots,\alpha$)</td>
</tr>
<tr>
<td>$n$</td>
<td>number of criteria</td>
</tr>
<tr>
<td>$X_i$</td>
<td>client’s satisfaction according to the $i$-th criterion ($i=1,2,\ldots,n$)</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>number of satisfaction levels for the $i$-th criterion</td>
</tr>
<tr>
<td>$x_i^k$</td>
<td>the $k$-th satisfaction level of the $i$-th criterion ($k=1,2,\ldots,\alpha_i$)</td>
</tr>
<tr>
<td>$Y^*$</td>
<td>value function of $Y$</td>
</tr>
<tr>
<td>$y_m^m$</td>
<td>value of the $y_m^m$ satisfaction level</td>
</tr>
<tr>
<td>$X_i^*$</td>
<td>value function of $X_i$</td>
</tr>
<tr>
<td>$x_i^k$</td>
<td>value of the $x_i^k$ satisfaction level</td>
</tr>
<tr>
<td>Customer</td>
<td>Global satisfaction</td>
</tr>
<tr>
<td>----------</td>
<td>---------------------</td>
</tr>
<tr>
<td>1</td>
<td>Satisfied</td>
</tr>
<tr>
<td>2</td>
<td>Dissatisfied</td>
</tr>
<tr>
<td>3</td>
<td>Very satisfied</td>
</tr>
<tr>
<td>4</td>
<td>Satisfied</td>
</tr>
<tr>
<td>5</td>
<td>Dissatisfied</td>
</tr>
<tr>
<td>6</td>
<td>Very satisfied</td>
</tr>
<tr>
<td>7</td>
<td>Satisfied</td>
</tr>
<tr>
<td>8</td>
<td>Satisfied</td>
</tr>
<tr>
<td>9</td>
<td>Satisfied</td>
</tr>
<tr>
<td>10</td>
<td>Dissatisfied</td>
</tr>
<tr>
<td>11</td>
<td>Satisfied</td>
</tr>
<tr>
<td>12</td>
<td>Dissatisfied</td>
</tr>
<tr>
<td>13</td>
<td>Very satisfied</td>
</tr>
<tr>
<td>14</td>
<td>Satisfied</td>
</tr>
<tr>
<td>15</td>
<td>Dissatisfied</td>
</tr>
<tr>
<td>16</td>
<td>Very satisfied</td>
</tr>
<tr>
<td>17</td>
<td>Very satisfied</td>
</tr>
<tr>
<td>18</td>
<td>Very satisfied</td>
</tr>
<tr>
<td>19</td>
<td>Satisfied</td>
</tr>
<tr>
<td>20</td>
<td>Dissatisfied</td>
</tr>
<tr>
<td>Variable</td>
<td>Value</td>
</tr>
<tr>
<td>----------</td>
<td>-------</td>
</tr>
<tr>
<td>$w_{11}$</td>
<td>0</td>
</tr>
<tr>
<td>$w_{12}$</td>
<td>25</td>
</tr>
<tr>
<td>$w_{21}$</td>
<td>25</td>
</tr>
<tr>
<td>$w_{22}$</td>
<td>25</td>
</tr>
</tbody>
</table>

Optimal value of the objective function $F^* = 0$
<table>
<thead>
<tr>
<th></th>
<th>$w_{i1}$</th>
<th>$w_{i2}$</th>
<th>$w_{21}$</th>
<th>$w_{22}$</th>
<th>$w_{31}$</th>
<th>$w_{32}$</th>
<th>$z_1$</th>
<th>$z_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>max $b_1$</td>
<td>10.00</td>
<td>22.50</td>
<td>22.50</td>
<td>22.50</td>
<td>22.50</td>
<td>0.00</td>
<td>55.00</td>
<td>45.00</td>
</tr>
<tr>
<td>max $b_2$</td>
<td>0.00</td>
<td>23.75</td>
<td>23.75</td>
<td>28.75</td>
<td>23.75</td>
<td>0.00</td>
<td>47.50</td>
<td>52.50</td>
</tr>
<tr>
<td>max $b_3$</td>
<td>0.00</td>
<td>20.00</td>
<td>20.00</td>
<td>30.00</td>
<td>30.00</td>
<td>0.00</td>
<td>50.00</td>
<td>50.00</td>
</tr>
<tr>
<td>Average</td>
<td>3.33</td>
<td>22.08</td>
<td>22.08</td>
<td>27.08</td>
<td>25.42</td>
<td>0.00</td>
<td>50.83</td>
<td>49.17</td>
</tr>
</tbody>
</table>
Table 5: Main results for the numerical example

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Weight</th>
<th>Average satisfaction index</th>
<th>Average demanding index</th>
<th>Average improvement index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product</td>
<td>25.42%</td>
<td>53.28%</td>
<td>0.74</td>
<td>0.12</td>
</tr>
<tr>
<td>Purchase process</td>
<td>49.17%</td>
<td>46.74%</td>
<td>0.10</td>
<td>0.26</td>
</tr>
<tr>
<td>Additional service</td>
<td>25.42%</td>
<td>55.00%</td>
<td>-1.00</td>
<td>0.11</td>
</tr>
<tr>
<td>Global satisfaction</td>
<td>-</td>
<td>50.33%</td>
<td>-0.02</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 6: Variations during the post-optimality analysis

<table>
<thead>
<tr>
<th>Weight</th>
<th>Min value</th>
<th>Max value</th>
<th>Estimated added value curve</th>
<th>Min value</th>
<th>Max value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>32.5%</td>
<td>20.0%</td>
<td>$y^1$</td>
<td>3.33</td>
<td>0.00</td>
</tr>
<tr>
<td>$b_2$</td>
<td>52.5%</td>
<td>45.0%</td>
<td>$y^2$</td>
<td>52.50</td>
<td>47.50</td>
</tr>
<tr>
<td>$b_3$</td>
<td>30.0%</td>
<td>22.5%</td>
<td>$y^3$</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>
Table 7: Post-optimality results for the numerical example (Generalised MUSA method)

<table>
<thead>
<tr>
<th></th>
<th>$w_{11}$</th>
<th>$w_{12}$</th>
<th>$w_{21}$</th>
<th>$w_{22}$</th>
<th>$w_{31}$</th>
<th>$w_{32}$</th>
<th>$z_1$</th>
<th>$z_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>max $b_1$</td>
<td>4.00</td>
<td>23.00</td>
<td>25.00</td>
<td>23.00</td>
<td>23.00</td>
<td>2.00</td>
<td>52.00</td>
<td>48.00</td>
</tr>
<tr>
<td>max $b_2$</td>
<td>2.00</td>
<td>23.25</td>
<td>25.25</td>
<td>24.25</td>
<td>23.25</td>
<td>2.00</td>
<td>50.50</td>
<td>49.50</td>
</tr>
<tr>
<td>max $b_3$</td>
<td>2.00</td>
<td>22.50</td>
<td>24.50</td>
<td>24.50</td>
<td>24.50</td>
<td>2.00</td>
<td>51.00</td>
<td>49.00</td>
</tr>
<tr>
<td>Average</td>
<td>2.67</td>
<td>22.92</td>
<td>24.92</td>
<td>23.92</td>
<td>23.58</td>
<td>2.00</td>
<td>51.17</td>
<td>48.83</td>
</tr>
</tbody>
</table>
Table 8: Comparison analysis of the post-optimality analysis results

<table>
<thead>
<tr>
<th></th>
<th>Basic MUSA method ( (\gamma = \gamma_i = 0) )</th>
<th>Generalised MUSA method ( (\gamma = \gamma_i = 2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( b_1 )</td>
<td>( b_2 )</td>
</tr>
<tr>
<td>max ( b_1 )</td>
<td>32.50</td>
<td>45.00</td>
</tr>
<tr>
<td>max ( b_2 )</td>
<td>23.75</td>
<td>52.50</td>
</tr>
<tr>
<td>max ( b_3 )</td>
<td>20.00</td>
<td>50.00</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>5.24</td>
<td>3.12</td>
</tr>
</tbody>
</table>
Table 9: Additional variables of the MUSA method (2 criteria levels)

\[ n_i \]: number of subcriteria for the \( i \)-th criterion

\[ X_{iq} \]: client’s satisfaction according to the \( q \)-th subcriterion of the \( i \)-th criterion (\( q=1,2,\ldots,n_i, \ i=1,2,\ldots,n \))

\[ \alpha_{iq} \]: number of satisfaction levels for the \( q \)-th subcriterion of the \( i \)-th criterion

\[ x_{iq}^k \]: the \( k \)-th satisfaction level for the \( q \)-th subcriterion of the \( i \)-th criterion (\( k=1, 2, \ldots, \alpha_{iq} \))

\[ X_{iq}^* \]: value function of \( X_{iq} \)

\[ x_{iq}^{*k} \]: value of the \( x_{iq}^k \) satisfaction level

\[ b_{iq} \]: weight for the \( q \)-th subcriterion of the \( i \)-th criterion
Table 10: Summarised results of alternative post-optimality analysis approaches

<table>
<thead>
<tr>
<th></th>
<th>Basic MUSA method ($\gamma = \gamma_i = 0$)</th>
<th>Generalised MUSA method ($\gamma = \gamma_i = 2$)</th>
<th>MUSA I</th>
<th>MUSA II</th>
<th>MUSA III</th>
<th>MUSA IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>25.42</td>
<td>25.58</td>
<td>26.46</td>
<td>26.21</td>
<td>26.51</td>
<td>25.00</td>
</tr>
<tr>
<td>$b_2$</td>
<td>49.17</td>
<td>48.83</td>
<td>48.75</td>
<td>49.25</td>
<td>49.06</td>
<td>50.00</td>
</tr>
<tr>
<td>$b_3$</td>
<td>25.42</td>
<td>25.58</td>
<td>24.79</td>
<td>24.54</td>
<td>24.43</td>
<td>25.00</td>
</tr>
</tbody>
</table>
Figure 1: Error variables for the $j$-th customer
Nutrality refers to the demanding index.

Neutral customers

Demanding customers

Non-demanding customers

Figure 2: Value functions with different demanding levels
Figure 3: Action diagram (Customers Satisfaction Council, 1995)
Figure 4: Improvement diagram
### Predicted global satisfaction level

<table>
<thead>
<tr>
<th>$\tilde{y}^1$</th>
<th>$\tilde{y}^2$</th>
<th>$\tilde{y}^i$</th>
<th>$\tilde{y}^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{i1}$ $R_{i1}$</td>
<td>$N_{i2}$ $R_{i2}$</td>
<td>...</td>
<td>$N_{i\alpha}$ $R_{i\alpha}$</td>
</tr>
<tr>
<td>$C_{i1}$ $C_{i2}$</td>
<td>...</td>
<td>...</td>
<td>$C_{i\alpha}$ $C_{i\alpha}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\tilde{y}^2$</th>
<th>$\tilde{y}^2$</th>
<th>$\tilde{y}^i$</th>
<th>$\tilde{y}^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{j1}$ $R_{j1}$</td>
<td>$N_{j2}$ $R_{j2}$</td>
<td>...</td>
<td>$N_{j\alpha}$ $R_{j\alpha}$</td>
</tr>
<tr>
<td>$C_{j1}$ $C_{j2}$</td>
<td>...</td>
<td>...</td>
<td>$C_{j\alpha}$ $C_{j\alpha}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\tilde{y}^i$</th>
<th>$\tilde{y}^i$</th>
<th>$\tilde{y}^i$</th>
<th>$\tilde{y}^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{k1}$ $R_{k1}$</td>
<td>$N_{k2}$ $R_{k2}$</td>
<td>...</td>
<td>$N_{k\alpha}$ $R_{k\alpha}$</td>
</tr>
<tr>
<td>$C_{k1}$ $C_{k2}$</td>
<td>...</td>
<td>...</td>
<td>$C_{k\alpha}$ $C_{k\alpha}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\tilde{y}^a$</th>
<th>$\tilde{y}^a$</th>
<th>$\tilde{y}^a$</th>
<th>$\tilde{y}^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\alpha1}$ $R_{\alpha1}$</td>
<td>$N_{\alpha2}$ $R_{\alpha2}$</td>
<td>...</td>
<td>$N_{\alpha\alpha}$ $R_{\alpha\alpha}$</td>
</tr>
<tr>
<td>$C_{\alpha1}$ $C_{\alpha2}$</td>
<td>...</td>
<td>...</td>
<td>$C_{\alpha\alpha}$ $C_{\alpha\alpha}$</td>
</tr>
</tbody>
</table>

**Figure 5:** Prediction table of global satisfaction
Figure 6: Value functions for the numerical example
Figure 7: Action diagrams for the numerical example
Figure 8: Improvement diagrams for the numerical example
Figure 9: Preference threshold for the value function $\Psi'$