Chapter 7

UTA METHODS

Yannis Siskos
University of Piraeus
Department of Informatics
80 Karaoli & Dimitriou Str.
18534 Piraeus – Greece
ysiskos@unipi.gr

Evangelos Grigoroudis, Nikolaos F. Matsatsinis
Technical University of Crete
Decision Support Systems Laboratory
University Campus, Kounoupidiana
73100 Chania – Greece
{vangelis,nikos}@ergasya.tuc.gr

Abstract: UTA methods refer to the philosophy of assessing a set of value or utility functions, assuming the axiomatic basis of MAUT and adopting the preference disaggregation principle. UTA methodology uses linear programming techniques in order to optimally infer additive value/utility functions, so that these functions are as consistent as possible with the global decision-maker's preferences (inference principle). The main objective of this chapter is to analytically present the UTA method and its variants and to summarize the progress made in this field. The historical background and the philosophy of the aggregation-disaggregation approach are firstly given in this chapter. The detailed presentation of the basic UTA algorithm is presented, including the discussion on the stability and sensitivity analyses. Several variants of the UTA method, which incorporate different forms of optimality criteria used in the LP formulation, are also discussed. The implementation of the UTA methods is illustrated by a general overview of UTA-based DSSs, as well as real-world decision-making applications. Finally, several potential future research developments of the UTA methodologies within the context of MCDA are discussed through this chapter.

Key words: UTA methods, Preference Disaggregation, Ordinal Regression, Additive Utility, Multicriteria Analysis

1. INTRODUCTION

1.1 General philosophy

In decision-making involving multiple criteria, the basic problem stated by analysts and decision-makers concerns the way that the final decision should be made. In many cases, however, this problem is posed in the opposite way: assuming that the decision is given, how is it possible to find the rational basis for the decision being made? Or equivalently, how is it possible to assess the decision–maker's preference model leading to exactly the same decision as the actual one or at least the most "similar" decision? The philosophy of preference disaggregation in multicriteria analysis is to assess/infer preference models from given preferential structures and to address decision-aiding activities through operational models within the aforementioned framework.

Under the term "multicriteria analysis" two basic approaches have been developed involving:

1. a set of methods or models enabling the aggregation of multiple evaluation criteria to choose one or more actions from a set *A* and
2. an activity of decision-aid to a well-defined decision-maker (individual, organization, etc.)

In both cases, the set *A* of potential actions (or objects, alternatives, decisions) is analyzed in terms of multiple criteria in order to model all the possible impacts, consequences or attributes related to the set *A*.

Roy (1985) outlines a general modeling methodology of decision-making problems, which includes four modeling steps starting with the definition of the set *A* and finishing with the activity of decision-aid, as follows:

– *Level 1*: Object of the decision, including the definition of the set of potential actions *A* and the determination of a problematic on *A*.
– *Level 2*: Modeling of a consistent family of criteria assuming that these criteria are non-decreasing value functions, exhaustive and non-redundant.
– *Level 3*: Development of a global preference model, to aggregate the marginal preferences on the criteria.
– *Level 4*: Decision-aid or decision support, based on the results of level 3 and the problematic of level 1.

In level 1, Roy (1985) distinguishes four reference problem statements, each of which does not necessarily preclude the others. These problematics can be employed separately, or in a complementary way, in all phases of the decision-making process. The four problematics are the following:

- *Problematic α*: Choosing one action from *A* (choice).
- *Problematic β*: Sorting the actions into pre-defined and preference-ordered categories (sorting).
- *Problematic γ*: Ranking the actions from the best one to the worst one (ranking).
- *Problematic δ*: Describing the actions in terms of their performances on the criteria (description).

In level 2, the modeling process must conclude on a consistent family of criteria $\{g_1, g_2, \ldots, g_n\}$. Each criterion is a non-decreasing real valued function defined on *A*, as follows:

$$g_i : A \rightarrow [g_{i*}, g_i^*] \subset \mathbb{R} \; / \; a \rightarrow g(a) \in \mathbb{R}$$

where $[g_{i*}, g_i^*]$ is the criterion evaluation scale, g_{i*} and g_i^* are the worst and the best level of the *i*-th criterion respectively, $g_i(a)$ is the evaluation or performance of action a on the *i*-th criterion and $\mathbf{g}(a)$ is the vector of performances of action a on the *n* criteria.

From the above definitions the following preferential situations can be determined:

$$\begin{cases} g_i(a) > g_i(b) \Leftrightarrow a \succ b & (a \text{ is preferred to } b) \\ g_i(a) = g_i(b) \Leftrightarrow a \sim b & (a \text{ is indifferent to } b) \end{cases}$$

So, having a weak-order preference structure on a set of actions, the problem is to adjust additive value or utility functions based on multiple criteria, in such a way that the resulting structure would be as consistent as possible with the initial structure. This principle underlies the disaggregation-aggregation approach presented in the next section.

This chapter is devoted to UTA methods, which are regression based approaches that have been developed as an alternative to multiattribute utility theory (MAUT). UTA methods not only adopt the aggregation-disaggregation principles, but they may also be considered as the main initiative and the most representative example of preference disaggregation theory. Another, more recent example of the preference disaggregation theory is the dominance-based rough set approach (DRSA) leading to decision rule preference model via inductive learning (see chapter 15 of this book).

1.2 The disaggregation-aggregation paradigm

In the traditional aggregation paradigm, the criteria aggregation model is known a priori, while the global preference is unknown. On the contrary, the philosophy of disaggregation involves the inference of preference models from given global preferences (*Figure 7-1*).

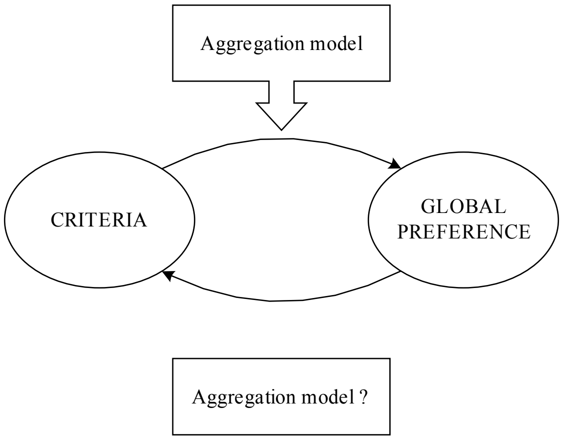

Figure 7-1. The aggregation and disaggregation paradigms in MCDA (Jacquet-Lagrèze and Siskos, 2001)

The disaggregation-aggregation approach (Jacquet-Lagrèze and Siskos, 1982; 2001; Siskos, 1980; Siskos and Yannacopoulos, 1985; Siskos *et al.*, 1993) aims at analyzing the behavior and the cognitive style of the Decision Maker (DM). Special iterative interactive procedures are used, where the components of the problem and the DM's global judgment policy are analyzed and then they are aggregated into a value system (*Figure 7-2*). The goal of this approach is to aid the DM to improve his/her knowledge about the decision situation and his/her way of preferring that entails a consistent decision to be achieved.

In order to use global preference given data, Jacquet-Lagrèze and Siskos (2001) note that the clarification of the DM's global preference necessitates the use of a set of reference actions A_R . Usually, this set could be:

1. a set of past decision alternatives (A_R : past actions),
2. a subset of decision actions, especially when A is large ($A_R \subset A$),
3. a set of fictitious actions, consisting of performances on the criteria, which can be easily judged by the decision-maker to perform global comparisons (A_R : fictitious actions).

In each of the above cases, the DM is asked to externalize and/or confirm his/her global preferences on the set A_R taking into account the performances of the reference actions on all criteria.

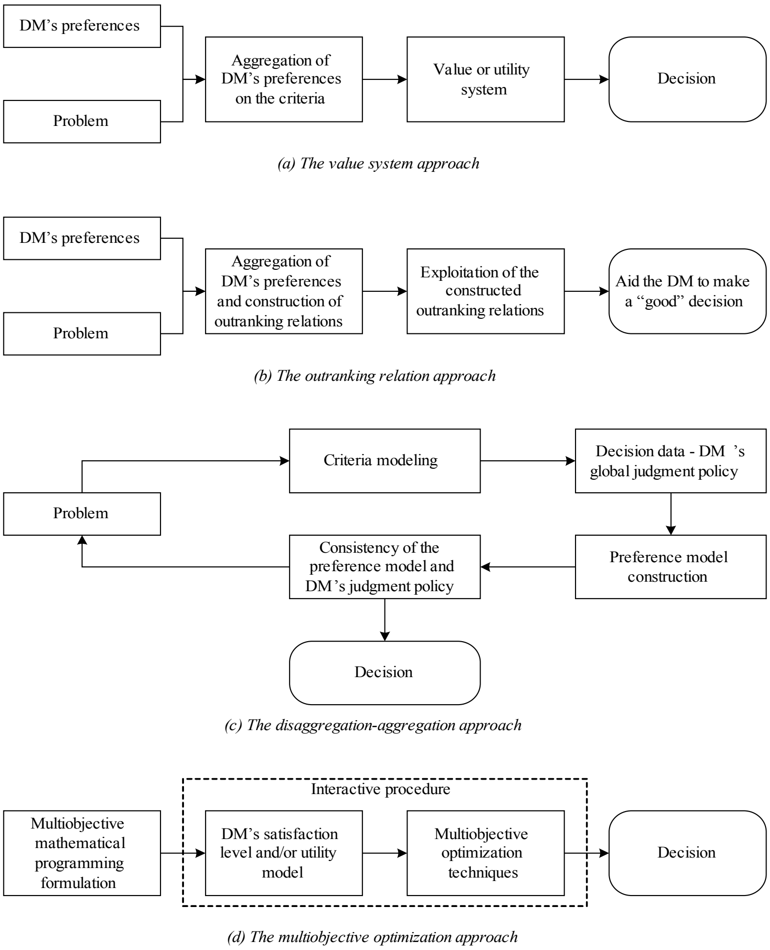

Figure 7-2. The disaggregation-aggregation approach vs. other MCDA approaches (Siskos and Spyridakos, 1999)

1.3 Historical background

The history of the disaggregation principle in multidimensional/ multicriteria analyses begins with the use of goal programming techniques, a special form of linear programming structure, in assessing/inferring preference/aggregation models or in developing linear or non–linear multidimensional regression analyses (Siskos, 1983).

Charnes *et al*. (1955) proposed a linear model of optimal estimation of executive compensation by analyzing or disaggregating pairwise comparisons and given measures (salaries); the model was estimated so that it could be as consistent as possible with the data from the goal programming point of view.

Karst (1958) minimized the sum of absolute deviations via goal programming in linear regression with one variable, while Wagner (1959) generalizes the Karst's model in the multiple regression case. Later Kelley (1958) proposed a similar model to minimize the Tchebycheff's criterion in linear regression.

Srinivasan and Shoker (1973) outlined the ORDREG ordinal regression model to assess a linear value function by disaggregating pairwise judgments. Freed and Glover (1981) proposed goal programming models to infer the weights of linear value functions in the frame of discriminant analysis (problematic β).

The research on handling ordinal criteria began with the studies of Young *et al*. (1976), and Jacquet-Lagrèze and Siskos (1978). The latter research refers to the presentation of the UTA method in the "Cahiers du LAMSADE" series and indicates the actual initiation of the development of disaggregation methods. Both research teams faced the same problem: to infer additive value functions by disaggregating a ranking of reference alternatives. Young *et al*. (1976) proposed alternating least squares techniques, without ensuring, however, that the additive value function is optimally consistent with the given ranking. In the case of the UTA method, optimality is ensured through linear programming techniques.

2. THE UTA METHOD

2.1 Principles and notation

The UTA (UTilités Additives) method proposed by Jacquet-Lagrèze and Siskos (1982) aims at inferring one or more additive value functions from a given ranking on a reference set A_R. The method uses special linear programming techniques to assess these functions so that the ranking(s) obtained through these functions on A_R is (are) as consistent as possible with the given one.

The criteria aggregation model in UTA is assumed to be an additive value function of the following form (Jacquet-Lagrèze and Siskos, 1982):

$$u(\mathbf{g}) = \sum_{i=1}^{n} p_i u_i(g_i)$$

subject to normalization constraints:

$$\begin{cases} \sum_{i=1}^{n} p_i = 1 \\ u_i(g_{i*}) = 0, \;\; u_i(g_i^*) = 1 \;\; \forall i = 1,2,\ldots,n \end{cases}$$

where u_i, $i = 1,2,\ldots,n$ are non-decreasing real valued functions, named marginal value or utility functions, which are normalized between 0 and 1, and p_i is the weight of u_i (*Figure 7-3*).

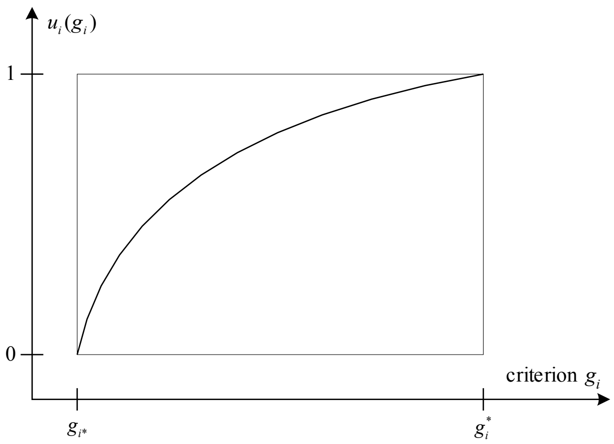

Figure 7-3. The normalized marginal value function

Both the marginal and the global value functions have the monotonicity property of the true criterion. For instance, in the case of the global value function the following properties hold:

$$\begin{cases} u[\mathbf{g}(a)] > u[\mathbf{g}(b)] \Leftrightarrow a \succ b \;\; \text{(preference)} \\ u[\mathbf{g}(a)] = u[\mathbf{g}(b)] \Leftrightarrow a \sim b \;\; \text{(indifference)} \end{cases}$$

The UTA method infers an unweighted form of the additive value function, equivalent to the form defined from relations (7-3) and (7-4), as follows:

$$u(\mathbf{g}) = \sum_{i=1}^{n} u_i(g_i)$$

subject to normalization constraints:

$$\begin{cases} \sum_{i=1}^{n} u_i(g_i^*) = 1 \\ u_i(g_{i*}) = 0 \quad \forall i = 1, 2, \ldots, n \end{cases}$$

Of course, the existence of such a preference model assumes the preferential independence of the criteria for the DM (Keeney and Raiffa, 1976), while other conditions for additivity have been proposed by Fishburn (1966, 1967). This assumption does not pose significant problems in a posteriori analyses such as disaggregation analyses.

2.2 Development of the UTA method

On the basis of the additive model (7-6)-(7-7) and taking into account the preference conditions (7-5), the value of each alternative $a \in A_R$ may be written as:

$$u'[\mathbf{g}(a)] = \sum_{i=1}^{n} u_i[g_i(a)] + \sigma(a) \quad \forall a \in A_R$$

where $\sigma(a)$ is a potential error relative to $u'[\mathbf{g}(a)]$.

Moreover, in order to estimate the corresponding marginal value functions in a piecewise linear form, Jacquet-Lagrèze and Siskos (1982) propose the use of linear interpolation. For each criterion, the interval $[g_{i*}, g_i^*]$ is cut into $(\alpha_i - 1)$ equal intervals, and thus the end points g_i^j are given by the formula:

$$g_i^j = g_{i*} + \frac{j-1}{\alpha_i - 1}\left(g_i^* - g_{i*}\right) \quad \forall j = 1, 2, \ldots, \alpha_i$$

The marginal value of an action a is approximated by a linear interpolation, and thus, for $g_i(a) \in [g_i^j, g_i^{j+1}]$

$$u_i[g_i(a)] = u_i(g_i^j) + \frac{g_i(a) - g_i^j}{g_i^{j+1} - g_i^j}\left[u_i(g_i^{j+1}) - u_i(g_i^j)\right]$$

The set of reference actions $A_R = \{a_1, a_2, \ldots a_m\}$ is also "rearranged" in such a way that a_1 is the head of the ranking and a_m its tail. Since the ranking has the form of a weak order R, for each pair of consecutive actions (a_k, a_{k+1}) it holds either $a_k \hbar a_{k+1}$ (preference) or $a_k : a_{k+1}$ (indifference). Thus, if

$$\Delta(a_k, a_{k+1}) = u'[(\mathbf{g}(a_k)] - u'[(\mathbf{g}(a_{k+1})]$$

then one of the following holds:

$$\begin{cases} \Delta(a_k, a_{k+1}) \geq \delta & \text{iff} \quad a_k \hbar a_{k+1} \\ \Delta(a_k, a_{k+1}) = 0 & \text{iff} \quad a_k : a_{k+1} \end{cases}$$

where δ is a small positive number so as to discriminate significantly two successive equivalence classes of R.

Taking into account the hypothesis on monotonicity of preferences, the marginal values $u_i(g_i)$ must satisfy the set of the following constraints:

$$u_i(g_i^{j+1}) - u_i(g_i^j) \geq s_i \quad \forall j = 1, 2, \ldots, \alpha_i - 1, \; i = 1, 2, \ldots, n$$

with $s_i \geq 0$ being indifference thresholds defined on each criterion g_i. Jacquet-Lagrèze and Siskos (1982) urge that it is not necessary to use these thresholds in the UTA model $(s_i = 0)$, but they can be useful in order to avoid phenomena such as $u_i(g_i^{j+1}) = u_i(g_i^j)$ when $g_i^{j+1} \hbar g_i^j$.

The marginal value functions are finally estimated by means of the following linear program (LP) with (7-6), (7-7), (7-12), (7-13) as constraints and with an objective function depending on the $\sigma(a)$ and indicating the amount of total deviation:

$$\begin{cases} [\min] F = \sum_{a \in A_R} \sigma(a) \\ \text{subject to} \\ \Delta(a_k, a_{k+1}) \geq \delta & \text{if } a_k \hbar a_{k+1} \\ \Delta(a_k, a_{k+1}) = 0 & \text{if } a_k : a_{k+1} \\ u_i(g_i^{j+1}) - u_i(g_i^j) \geq 0 & \forall i \text{ and } j \\ \sum_{i=1}^{n} u_i(g_i^*) = 1 \\ u_i(g_{i*}) = 0, \; u_i(g_i^j) \geq 0, \; \sigma(a) \geq 0 \quad \forall a \in A_R, \; \forall i \text{ and } j \end{cases}$$

The stability analysis of the results provided by LP (7-14) is considered as a post-optimality analysis problem. As Jacquet-Lagrèze and Siskos (1982) note, if the optimum $F^* = 0$, the polyhedron of admissible solutions for $u_i(g_i)$ is not empty and many value functions lead to a perfect representation of the weak order R. Even when the optimal value F^* is strictly positive, other solutions, less good for F, can improve other satisfactory criteria, like Kendall's τ.

As shown in *Figure 7-4*, the post-optimal solutions space is defined by the polyhedron:

$$\begin{cases} F \leq F^* + k(F^*) \\ \text{all the constraints of LP (7-14)} \end{cases}$$

where $k(F^*)$ is a positive threshold which is a small proportion of F^*.

The algorithms which could be used to explore the polyhedron (7-15) are branch and bound methods, like reverse simplex method (Van de Panne, 1975), or techniques dealing with the notion of the labyrinth in graph theory, such as Tarry's method (Charnes and Cooper, 1961), or the method of Manas and Nedoma (1968). Jacquet-Lagrèze and Siskos (1982), in the original UTA method, propose the partial exploration of polyhedron (7-15) by solving the following LPs:

$$\begin{cases} [\min] u_i(g_i^*) \\ \text{in} \\ \text{polyhedron (7-15)} \end{cases} \quad \text{and} \quad \begin{cases} [\max] u_i(g_i^*) \\ \text{in} \\ \text{polyhedron (7-15)} \end{cases} \qquad \forall i = 1, 2, \ldots, n$$

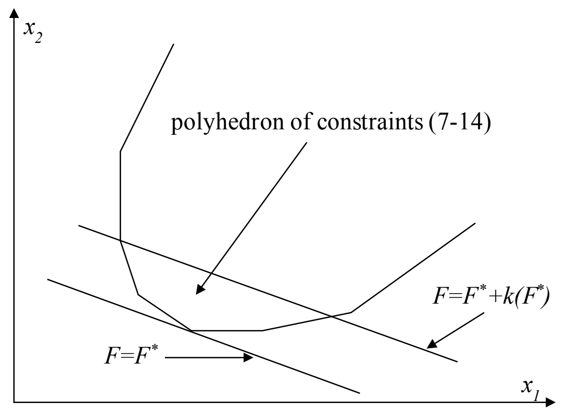

Figure 7-4. Post-optimality analysis (Jacquet-Lagrèze and Siskos, 1982)

The average of the previous LPs may be considered as the final solution of the problem. In case of instability, a large variation of the provided solutions appears, and this average solution is less representative. In any case, the solutions of the above LPs give the internal variation of the weight of all criteria g_i, and consequently give an idea of the importance of these criteria in the DM's preference system.

2.3 The UTASTAR algorithm

The UTASTAR method proposed by Siskos and Yannacopoulos (1985) is an improved version of the original UTA model presented in the previous section. In the original version of UTA (Jacquet-Lagrèze and Siskos, 1982), for each packed action $a \in A_R$, a single error $\sigma(a)$ is introduced to be minimized. This error function is not sufficient to minimize completely the dispersion of points all around the monotone curve of *Figure 7-5*. The problem is posed by points situated on the right of the curve, from which it would be suitable to subtract an amount of value/utility and not increase the values/utilities of the others.

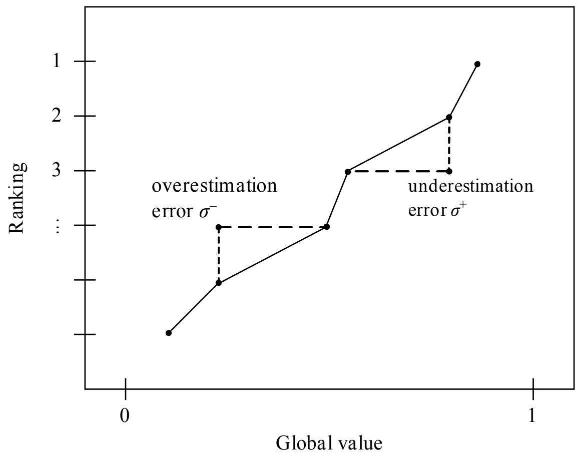

Figure 7-5. Ordinal regression curve (ranking versus global value)

In UTASTAR method, Siskos and Yannacopoulos (1985) introduced a double positive error function, so that formula (7-8) becomes:

$$u'[\mathbf{g}(a)] = \sum_{i=1}^{n} u_i[g_i(a)] - \sigma^+(a) + \sigma^-(a) \quad \forall a \in A_R$$

where σ^+ and σ^- are the overestimation and the underestimation error respectively.

Moreover, another important modification concerns the monotonicity constraints of the criteria, which are taken into account through the transformations of the variables:

$$w_{ij} = u_i(g_i^{j+1}) - u_i(g_i^j) \geq 0 \quad \forall i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,\alpha_i - 1$$

and thus, the monotonicity conditions (7-13) can be replaced by the non-negative constraints for the variables w_{ij} (for $s_i = 0$).

Consequently, the UTASTAR algorithm may be summarized in the following steps:

Step 1:

Express the global value of reference actions $u[\mathbf{g}(a_k)]$, $k = 1,2,\ldots,m$, first in terms of marginal values $u_i(g_i)$, and then in terms of variables w_{ij} according to the formula (7-17), by means of the following expressions:

$$\begin{cases} u_i(g_i^1) = 0 & \forall i = 1,2,\ldots,n \\ u_i(g_i^j) = \sum_{t=1}^{j-1} w_{it} & \forall i = 1,2,\ldots,n \text{ and } j = 2,3,\ldots,\alpha_i - 1 \end{cases}$$

Step 2:

Introduce two error functions σ^+ and σ^- on A_R by writing for each pair of consecutive actions in the ranking the analytic expressions:

$$\Delta(a_k, a_{k+1}) = u[\mathbf{g}(a_k)] - \sigma^+(a_k) + \sigma^-(a_k) \\ - u[\mathbf{g}(a_{k+1})] + \sigma^+(a_{k+1}) - \sigma^-(a_{k+1})$$

Step 3:

Solve the linear program:

$$\begin{cases} [\min] z = \sum_{k=1}^{m} [\sigma^+(a_k) + \sigma^-(a_k)] \\ \text{subject to} \\ \left.\begin{array}{l} \Delta(a_k, a_{k+1}) \geq \delta \quad \text{if} \quad a_k \hbar a_{k+1} \\ \Delta(a_k, a_{k+1}) = 0 \quad \text{if} \quad a_k : a_{k+1} \end{array}\right\} \forall k \\ \sum_{i=1}^{n} \sum_{j=1}^{\alpha_i - 1} w_{ij} = 1 \\ w_{ij} \geq 0,\ \sigma^+(a_k) \geq 0,\ \sigma^-(a_k) \geq 0 \quad \forall i, j \text{ and } k \end{cases}$$

with δ being a small positive number.

Step 4:

Test the existence of multiple or near optimal solutions of the linear program (7-21) (stability analysis); in case of non uniqueness, find the mean additive value function of those (near) optimal solutions which maximize the objective functions:

$$u_i(g_i^*) = \sum_{j=1}^{\alpha_i - 1} w_{ij} \quad \forall i = 1,2,\ldots,n$$

on the polyhedron of the constraints of the LP (7-21) bounded by the new constraint:

$$\sum_{k=1}^{m} \left[\sigma^+(a_k) + \sigma^-(a_k)\right] \leq z^* + \varepsilon$$

where z^* is the optimal value of the LP in step 3 and ε a very small positive number.

A comparison analysis between UTA and UTASTAR algorithms is presented by Siskos and Yannacopoulos (1985) through a variety of experimental data. UTASTAR method has provided better results concerning a number of comparison indicators, like:

1. The number of the necessary simplex iterations for arriving at the optimal solution.
2. The Kendall's τ between the initial weak order and the one produced by the estimated model.
3. The minimized criterion z (sum of errors) taken as the indicator of dispersion of the observations.

2.4 A numerical example

The implementation of the UTASTAR algorithm is illustrated by a practical example presented by Siskos and Yannacopoulos (1985). The problem concerns a DM who wishes to analyze the choice of transportation means during the peak hours (home-work place). Suppose that the DM is interested only in the following three criteria:

1. price (in monetary units),
2. time of journey (in minutes), and
3. comfort (possibility to have a seat).

The evaluation in terms of the previous criteria is presented in *Table 7-1*, where it should be noted that the following qualitative scale has been used for the comfort criterion: 0 (no chance of seating), + (little chance of seating) ++ (great chance of finding a seating place), and +++ (seat assured). Also, the last column of *Table 7-1* shows the DM's ranking with respect to the five alternative means of transportation.

Table 7-1. Criteria values and ranking of the DM

Means of transportation	Price (mu)	Time (min)	Comfort	Ranking of the DM
RER	3	10	+	1
METRO (1)	4	20	++	2
METRO (2)	2	20	0	2
BUS	6	40	0	3
TAXI	30	30	+++	4

The first step of UTASTAR, as presented in the previous section, consists of making explicit the utilities of the five alternatives. For this reason the following scales have been chosen:

$$\left[g_{1*}, g_1^*\right] = [30, 16, 2]$$

$$\left[g_{2*}, g_2^*\right] = [40, 30, 20, 10]$$

$$\left[g_{3*}, g_3^*\right] = [0, +, ++, +++]$$

Using linear interpolation for the criterion g_1 according to formula (7-10), the value of each alternative may be written as:

$$u[\mathbf{g}(\text{RER})] = 0.07u_1(16) + 0.93u_1(2) + u_2(10) + u_3(+)$$
$$u[\mathbf{g}(\text{METRO1})] = 0.14u_1(16) + 0.86u_1(2) + u_2(20) + u_3(++)$$
$$u[\mathbf{g}(\text{METRO2})] = u_1(2) + u_2(20) + u_3(0)$$
$$= u_1(2) + u_2(20)$$
$$u[\mathbf{g}(\text{BUS})] = 0.29u_1(16) + 0.71u_1(2) + u_2(40) + u_3(0)$$
$$= 0.29u_1(16) + 0.71u_1(2)$$
$$u[\mathbf{g}(\text{TAXI})] = u_1(30) + u_2(30) + u_3(+++)$$
$$= u_2(30) + u_3(+++)$$

where the following normalization conditions for the marginal value functions have been used: $u_1(30) = u_2(40) = u_3(0) = 0$.

Also, according to formula (7-19), the global value of the alternatives may be expressed in terms of the variables w_{ij}:

$$u[\mathbf{g}(\text{RER})] = w_{11} + 0.93w_{12} + w_{21} + w_{22} + w_{23} + w_{31}$$
$$u[\mathbf{g}(\text{METRO1})] = w_{11} + 0.86w_{12} + w_{21} + w_{22} + w_{31} + w_{32}$$
$$u[\mathbf{g}(\text{METRO2})] = w_{11} + w_{12} + w_{21} + w_{22}$$
$$u[\mathbf{g}(\text{BUS})] = w_{11} + 0.71w_{12}$$
$$u[\mathbf{g}(\text{TAXI})] = w_{21} + w_{31} + w_{32} + w_{33}$$

According to the second step of the UTASTAR algorithm, the following expressions are written, for each pair of consecutive actions in the ranking:

$$\Delta(\text{RER},\text{METRO1}) = 0.07w_{12} + w_{23} - w_{32}$$
$$-\sigma^+_{\text{RER}} + \sigma^-_{\text{RER}} + \sigma^+_{\text{METRO1}} - \sigma^-_{\text{METRO1}}$$
$$\Delta(\text{METRO1},\text{METRO2}) = -0.14w_{12} + w_{31} + w_{32}$$
$$-\sigma^+_{\text{METRO1}} + \sigma^-_{\text{METRO1}} + \sigma^+_{\text{METRO2}} - \sigma^-_{\text{METRO2}}$$
$$\Delta(\text{METRO2},\text{BUS}) = 0.29w_{12} + w_{21} + w_{22}$$
$$-\sigma^+_{\text{METRO2}} + \sigma^-_{\text{METRO2}} + \sigma^+_{\text{BUS}} - \sigma^-_{\text{BUS}}$$
$$\Delta(\text{BUS},\text{TAXI}) = w_{11} + 0.71w_{12} - w_{21} - w_{31} - w_{32} - w_{33}$$
$$-\sigma^+_{\text{BUS}} + \sigma^-_{\text{BUS}} + \sigma^+_{\text{TAXI}} - \sigma^-_{\text{TAXI}}$$

Based on the aforementioned expression, a linear program according to (7-21) is formulated, with $\delta = 0.05$ (see *Table 7-2*). An optimal solution is: $w_{11} = 0.5$, $w_{21} = 0.05$, $w_{23} = 0.05$, $w_{33} = 0.4$ with $[\min] z = z^* = 0$. This solution corresponds to the marginal value functions presented in *Table 7-3* and produces a ranking which is consistent with the DM's initial weak order.

Table 7-2. Initial linear programming formulation

w_{11}	w_{12}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}	variables σ^+ and σ^-											RHS
0	0.07	0	0	1	0	-1	0	-1	1	1	-1	0	0	0	0	0	0	≥	0.05

w_{11}	w_{12}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}	variables σ^+ and σ^-											RHS
0	-0.14	0	0	0	1	1	0	0	0	-1	1	1	-1	0	0	0	0	=	0
0	0.29	1	1	0	0	0	0	0	0	0	0	-1	1	1	-1	0	0	≥	0.05
1	0.71	-1	0	0	-1	-1	-1	0	0	0	0	0	0	-1	1	1	-1	≥	0.05
1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	=	1
1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0		z

It should be emphasized that this solution is not unique. Through post-optimality analysis (step 4), the UTASTAR algorithm searches for multiple optimal solutions, or more generally, for near optimal solutions corresponding to error values between z^* and $z^*+\varepsilon$. For this reason, the error objective should be transformed to a constraint of the type (7-23).

Table 7-3. Marginal value functions (initial solution)

Price	Time	Comfort
$u_1(30)$ = 0.000	$u_2(40)$ = 0.000	$u_3(0)$ = 0.000
$u_1(16)$ = 0.500	$u_2(30)$ = 0.050	$u_3(+)$ = 0.000
$u_1(2)$ = 0.500	$u_2(20)$ = 0.050	$u_3(++)$ = 0.000
	$u_2(10)$ = 0.100	$u_3(+++)$ = 0.400

In the presented numerical example, the initial linear program has multiple optimal solutions, since $z^*=0$. Thus, in the post-optimality analysis step, the algorithm searches for more characteristic solutions, which maximize the expressions (7-22), i.e. the weights of each criterion. Furthermore, in this particular case we have:

$$z^*=0 \Leftrightarrow \sigma^+(a_k)=\sigma^-(a_k)=0 \ \ \forall k$$

so the error variables may be excluded from the linear programs of the post-optimality analysis. *Table 7-4* presents the formulation of the linear programs that have to be solved during this step.

Table 7-4. Linear programming formulation (post-optimality analysis)

w_{11}	w_{12}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}		RHS
0	0.07	0	0	1	0	-1	0	≥	0.05
0	-0.14	0	0	0	1	1	0	=	0
0	0.29	1	1	0	0	0	0	≥	0.05
1	0.71	-1	0	0	-1	-1	-1	≥	0.05
1	1	1	1	1	1	1	1	=	1
1	1	0	0	0	0	0	0	[max] $u_1(g_1^*)$	
0	0	1	1	1	0	0	0	[max] $u_2(g_2^*)$	
0	0	0	0	0	1	1	1	[max] $u_3(g_3^*)$	

The solutions obtained during post-optimality analysis are presented in *Table 7-5*. The average of these three solutions is also calculated in the last row of *Table 7-5*. This centroid is taken as a unique utility function, provided that it is considered as a more representative solution of this particular problem.

Table 7-5. Post-optimality analysis and final solution

	w_{11}	w_{12}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}
[max] $u_1(g_1^*)$	0.7625	0.175	0	0	0.0375	0.025	0	0
[max] $u_2(g_2^*)$	0.05	0	0	0.05	0.9	0	0	0
[max] $u_3(g_3^*)$	0.3562	0.175	0	0	0.0375	0.025	0	0.4063
Average	0.3896	0.1167	0	0.0167	0.3250	0.0167	0	0.1354

This final solution corresponds to the marginal value functions presented in *Table 7-6*. Also, the utilities for each alternative are calculated as follows:

$$u[\mathbf{g}(\text{RER})] = 0.856$$
$$u[\mathbf{g}(\text{METRO1})] = 0.523$$
$$u[\mathbf{g}(\text{METRO2})] = 0.523$$
$$u[\mathbf{g}(\text{BUS})] = 0.473$$
$$u[\mathbf{g}(\text{TAXI})] = 0.152$$

where it is obvious that these values are consistent with the DM's weak order.

Table 7-6. Marginal value functions (final solution)

Price	Time	Comfort
$u_1(30)$ = 0.000	$u_2(40)$ = 0.000	$u_3(0)$ = 0.000
$u_1(16)$ = 0.390	$u_2(30)$ = 0.000	$u_3(+)$ = 0.017
$u_1(2)$ = 0.506	$u_2(20)$ = 0.017	$u_3(++)$ = 0.017
	$u_2(10)$ = 0.342	$u_3(+++)$ = 0.152

These marginal utilities may be normalized by dividing every value $u_i(g_i^j)$ by $u_i(g_i^*)$. In this case the additive utility can be written as:

$$u(\mathbf{g}) = 0.506u_1(g_1) + 0.342u_2(g_2) + 0.152u_3(g_3)$$

where the normalized marginal value functions are presented in *Figure 7.6*.

3. VARIANTS OF THE UTA METHOD

3.1 Alternative optimality criteria

Several variants of the UTA method have been developed, incorporating different forms of global preference or different forms of optimality criteria used in the linear programming formulation.

An extension of the UTA methods, where $u[\mathbf{g}(a)]$ is inferred from pairwise comparisons is proposed by Jacquet-Lagrèze and Siskos (1982). This subjective preference obtained by pairwise judgments is most often not transitive, and thus, the modified model may be written as in the following LP:

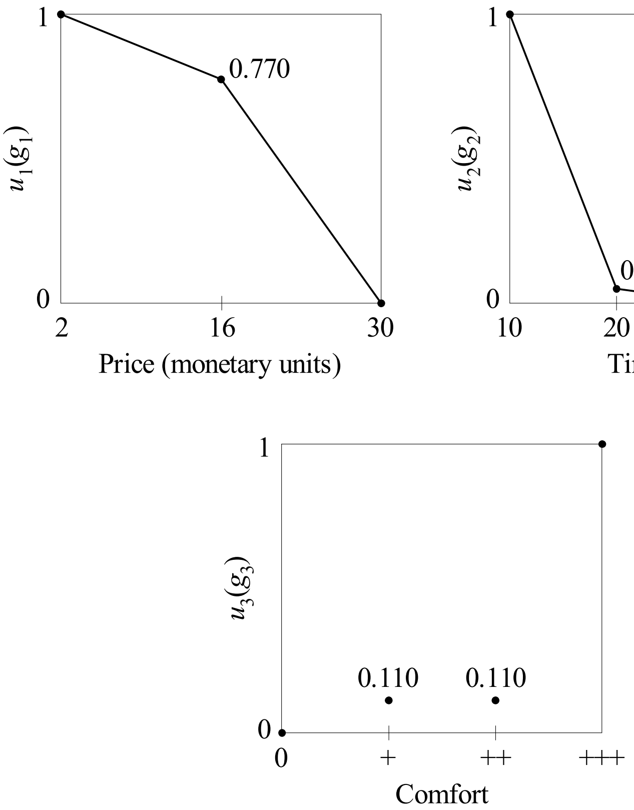

Figure 7-6. Normalized marginal value functions

$$\begin{cases}[\min]F = \sum\limits_{(a,b):a \succ b} \lambda_{ab} z_{ab} + \sum\limits_{(a,b):a \sim b} \lambda_{ab} z_{ba} \\ \text{subject to} \\ \sum\limits_{i=1}^{n} \left\{ u_i \left[g_i(a) \right] - u_i \left[g_i(b) \right] \right\} + z_{ab} \geq 0 \qquad \text{if } a \succ b \\ \sum\limits_{i=1}^{n} \left\{ u_i \left[g_i(a) \right] - u_i \left[g_i(b) \right] \right\} + z_{ab} - z_{ba} = 0 \quad \text{if } a \sim b \ (\Rightarrow b \sim a) \\ u_i(g_i^{j+1}) - u_i(g_i^{j}) \geq s_i \qquad \forall i, j \\ \sum\limits_{i=1}^{n} u_i(g_i^{*}) = 1 \\ u_i(g_{i*}) = 0,\ u_i(g_i^{j}) \geq 0,\ z_{ab} \geq 0 \quad \forall i, j \text{ and } (a,b) \in R \end{cases}$$

λ_{ab} being a non negative weight reflecting a degree of confidence in the judgment between a and b.

An alternative optimality criterion would be to minimize the number of violated pairs of an order R provided by the DM in ranking R' given by the model, which is equivalent to maximize Kendall's τ between the two rankings. This extension is given by the mixed integer LP (7-25), where $\gamma_{ab} = 0$ if $u[\mathbf{g}(a)] - u[\mathbf{g}(b)] \geq \delta$ for a pair $(a,b) \in R$ and the judgment is respected, otherwise $\gamma_{ab} = 1$ and the judgment is violated. Thus, the objective function in this LP represents the number of violated pairs in the overall preference aggregated by $u(\mathbf{g})$.

$$\begin{cases}[\min]F = \sum\limits_{(a,b)\in R} \gamma_{ab} \Leftrightarrow [\max]\tau(R, R') \\ \text{subject to} \\ \sum\limits_{i=1}^{n} \left\{ u_i \left[g_i(a) \right] - u_i \left[g_i(b) \right] \right\} + M \cdot \gamma_{ab} \geq \delta \quad \forall (a,b) \in R \\ u_i(g_i^{j+1}) - u_i(g_i^{j}) \geq s_i \qquad \forall i, j \\ \sum\limits_{i=1}^{n} u_i(g_i^{*}) = 1 \\ u_i(g_{i*}) = 0,\ u_i(g_i^{j}) \geq 0 \qquad \forall i, j \\ \gamma_{ab} = 0 \text{ or } 1 \qquad \forall (a,b) \in R \end{cases}$$

where M is a large number. Beuthe and Scanella (2001) propose to handle separately the preference and indifference judgments, and modify the previous LP using the constraints:

$$
\begin{cases}
\sum_{i=1}^{n}\left\{u_i[g_i(a)]-u_i[g_i(b)]\right\}+M\cdot\gamma_{ab}\geq\delta & \text{if } a \hbar b \\
\left.\begin{aligned}
&\sum_{i=1}^{n}\left\{u_i[g_i(a)]-u_i[g_i(b)]\right\}+M\cdot\gamma_{ab}\geq 0\\
&\sum_{i=1}^{n}\left\{u_i[g_i(a)]-u_i[g_i(b)]\right\}+M\cdot\gamma_{ba}\geq 0
\end{aligned}\right\} & \text{if } a : b
\end{cases}
$$

The assumption of monotonicity of preferences, in the context of separable value functions, means that the marginal values are monotonic functions of the criteria. This assumption, although widely used, is sometimes not applicable to real-world situations. One way to deal with non-monotonic preferences is to divide the range of the criteria into intervals, so that the preferences are monotonic in each interval, and then treat each interval separately (Keeney and Raiffa, 1976). In the same spirit, Despotis and Zopounidis (1993) present a variation of the UTASTAR method for the assessment of non-monotonic marginal value functions. In this model, the range of each criterion is divided into two intervals (see also *Figure 7-7*):

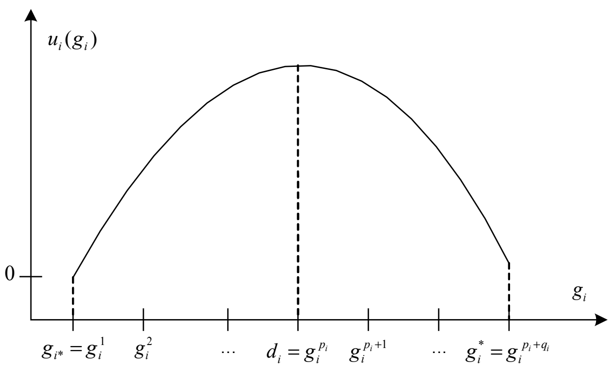

Figure 7-7. A non-monotonic partial utility function (Despotis and Zopounidis, 1993)

$$
\begin{cases}
G_i^1 = \left\{ g_{i*} = g_i^1, g_i^2, \ldots, g_i^{p_i} = d_i \right\} \\
G_i^2 = \left\{ d_i = g_i^{p_i}, g_i^{p_i+1}, \ldots, g_i^{p_i+q_i} = g_i^* \right\}
\end{cases}
$$

where d_i is the most desirable value of g_i, and the parameters p_i and q_i are determined according to the dispersion of the input data; of course it holds that $p_i + q_i = \alpha_i$. In this approach, the main modification concerns the assessment of the decision variables w_{ij} of the LP (7-21). Hence, formula (7-19) becomes:

$$u_i(g_i^j) = \begin{cases} \sum_{t=1}^{j-1} w_{it} & \text{if } 1 < j \le p_i \\ \sum_{t=1}^{p_i-1} w_{it} - \sum_{t=p_i}^{j-1} w_{it} & \text{if } p_i < j \le \alpha_i \end{cases}$$

while the conditions $u_i(g_i^1) = 0$ remain.

Another extension of the UTA methods refers to the intensity of the DM's preferences, similar to the context proposed by Srinivasan and Socker (1973). In this case, a series of constraints may be added during the LP formulation. For example, if the preference of alternative a over alternative b is stronger than the preference of b over c, then the following condition may be written:

$$\left[u'[\mathbf{g}(a)] - u'[\mathbf{g}(b)]\right] - \left[u'[\mathbf{g}(b)] - u'[\mathbf{g}(c)]\right] \ge \varphi$$

where $\varphi > 0$ is a measure of preference intensity and $u'(\mathbf{g})$ is given by formula (7-8). Thus, using formula (7-11), the following constraint should be added in LP (7-14):

$$\Delta(a,b) - \Delta(b,c) \ge \varphi$$

In general, if the DM wishes to expand these preferences to the whole set of alternatives, a minimum number of $m-2$ constraints of type (7-30) is required.

Despotis and Zopounidis (1993) consider the case where the DM ranks the alternatives using an explicit overall index I. Thus, formula (7-12) may be replaced by the following condition:

$$\Delta(a_k, a_{k+1}) = I_k - I_{k+1} \quad \forall k = 1,2,\dots,m-1$$

Besides the succession of the alternatives in the preference ranking, these constraints state that the difference of global value of any successive alternatives in the ranking should be consistent with the difference of their evaluation on the ratio scale.

In the same context, Oral and Ketanni (1989) propose the optimization of lexicographic criteria without discretisation of criteria scales G_i, where a ratio scale is used in order to express intensity of preferences.

Other variants of the UTA method concerning different forms of global preference are mainly focused on:

– additional properties of the assessed value functions, like concavity (Despotis and Zopounidis, 1993);
– construction of fuzzy outranking relations based on multiple value functions u provided by UTA's post-optimality analysis (Siskos, 1982).

The dimensions of the aforementioned UTA models affect the computational complexity of the formulated LPs. In most cases, as noted by Jacquet-Lagrèze and Siskos (1982), it is preferable to solve the dual LP sue to the structure of these LPs. *Table 7-7* summarizes the size of all LPs presented in the previous sections, where $|P|$ and $|I|$ denote the number of preference and indifference relations respectively, considering all possible pairwise comparisons in R. Also, it should be noted that LP (7-25) has $m(m-1)/2$ binary variables.

Table 7-7. LP size of UTA models

Model and optimality criterion	Constraints	Variables
UTA - Min sum of errors (LP 7-14)	$m+\sum_{i=1}^{n}(\alpha_i-1)$	$m+\sum_{i=1}^{n}(\alpha_i-1)$
UTASTAR - Min sum of errors (LP 7-21)	m	$2m+\sum_{i=1}^{n}(\alpha_i-1)$
UTA - Min sum of errors from pairwise judgments (LP 7-24)	$1+[m(m-1)/2]+\sum_{i=1}^{n}(\alpha_i-1)$	$\lvert P\rvert+2\lvert I\rvert+\sum_{i=1}^{n}(\alpha_i-1)$
UTA – Max Kendall's τ (LP 7-25)	$1+[m(m-1)/2]+\sum_{i=1}^{n}(\alpha_i-1)$	$[m(m-1)/2]+\sum_{i=1}^{n}(\alpha_i-1)$

3.2 Meta-UTA techniques

Other techniques named meta-UTA, aimed at the improvement of the value function with respect to near optimality analysis or to its exploitation for decision support.

Despotis *et al*. (1990) propose to minimize the dispersion of errors (Tchebycheff criterion) within the UTASTAR's step 4 (see section 2.3). In case of a strictly positive error $(z^*>0)$, the aim is to investigate the existence of near optimal solutions of the LP (7-21) which give rankings R' such that $\tau(R',R)>\tau(R^*,R)$, with R^* being the ranking corresponding to the optimal value functions. The experience with the model (cf. Despotis and Yannacopoulos, 1990) confirms that apart from the total error z^*, it is also the dispersion of the individual errors that is crucial for $\tau(R^*,R)$. Therefore, in the proposed post-optimality analysis, the difference between the maximum $(\sigma_{\max})$ and the minimum error is minimized. As far as the individual errors are non-negative, this requirement can be satisfied by minimizing the maximum individual error (the L_∞ norm) according to the following LP:

$$\begin{cases} [\min]\ \sigma_{\max} \\ \text{subject to} \\ \text{all the constraints of LP (7-21)} \\ \sum_{k=1}^{m} \left[\sigma^{+}(a_k) + \sigma^{-}(a_k) \right] \leq z^{*} + \varepsilon \\ \sigma_{\max} - \sigma^{+}(a_k) \geq 0 \\ \sigma_{\max} - \sigma^{-}(a_k) \geq 0 \qquad \forall k \\ \sigma_{\max} \geq 0 \end{cases}$$

With the incorporation of the model (7-29) in UTASTAR, the value function assessment process becomes a lexicographic optimization process. That is, the final solution is obtained by minimizing successively the L_1 and the L_∞ norms.

Another approach concerning meta-UTA techniques refers to the UTAMP models. Beuthe and Scanella (1996, 2001) note that the values given to parameters s and δ in the UTA and UTASTAR methods, respectively, influence the results as well as the predictive quality of the models. Hence, in the framework of the research by Srinivasan and Shocker (1973), they look for optimal values of s and/or δ in the case of positive errors $(z^{*} > 0)$, as well as when UTA gives a sum of error equal to zero $(z^{*} = 0)$.

In the post-optimality analysis step of UTASTAR (see section 2.3), UTAMP1 model maximizes δ, which is the minimum difference between the global value of two consecutive reference actions. The name of the model denotes that, on the basis of UTA, maximizing δ leads to better identification for the relations of preference between actions.

Beuthe and Scanella (1996) have also proposed to maximize the sum $(\delta + s)$ in order to stress not only the differences of utilities between actions, but also the differences between values at successive bounds. This more general approach was named UTAMP2. Note that s corresponds to the minimum of marginal value step w_{ij} in the UTASTAR algorithm. Although the simple addition of these parameters is legitimate since both of them are defined in the same value units, Beuthe and Scanella (2001) note that a weighted sum formula may also be considered.

3.3 Stochastic UTA method

Within the framework of multicriteria decision-aid under uncertainty, Siskos (1983) developed a specific version of UTA (Stochastic UTA), in which the aggregation model to infer from a reference ranking is an additive utility function of the form:

$$u(\boldsymbol{\delta}^a) = \sum_{i=1}^{n}\sum_{j=1}^{\alpha_i} \delta_i^a(g_i^j)u_i(g_i^j)$$

subject to normalization constraints (7-7), where δ_i^a is the distributional evaluation of action a on the *i*-th criterion, $\delta_i^a(g_i^j)$ is the probability that the performance of action a on the *i*-th criterion is g_i^j, $u_i(g_i^j)$ is the marginal value of the performance g_i^j, $\boldsymbol{\delta}^a$ is the vector of distributional evaluations of action a and $u(\boldsymbol{\delta}^a)$ is the global utility of action a (see also *Figure 7-8*).

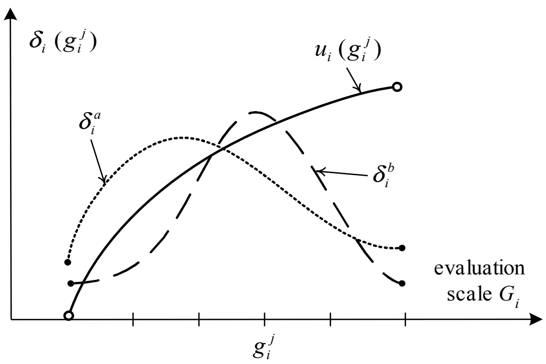

Figure 7-8. Distributional evaluation and marginal value function

This global utility is of the von Neumann-Morgenstern form (cf. Keeney, 1980), in the case of discrete g_i, where:

$$\sum_{j=1}^{\alpha_i} \delta_i^a(g_i^j) = 1$$

Of course, the additive utility function (7-30) has the same properties as the value function:

$$\begin{cases} u(\boldsymbol{\delta}^a) > u(\boldsymbol{\delta}^b) \Leftrightarrow a \hbar b & \text{(preference)} \\ u(\boldsymbol{\delta}^a) = u(\boldsymbol{\delta}^b) \Leftrightarrow a : b & \text{(indifference)} \end{cases}$$

Similarly to the cases of UTA and UTASTAR described in sections 2.2-2.3, the stochastic UTA method disaggregates a ranking of reference actions (Siskos and Assimakopoulos, 1989). The algorithmic procedure could be expressed in the following way:

Step 1:

Express the global expected utilities of reference actions $u(\boldsymbol{\delta}^{a_k})$, $k=1,2,\ldots,m$, in terms of variables:

$$w_{ij}=u_i(g_i^{j+1})-u_i(g_i^j)\geq 0$$

Step 2:

Introduce two error functions σ^+ and σ^- by writing the following expressions for each pair of consecutive actions in the ranking:

$$\Delta(a_k,a_{k+1})=u(\boldsymbol{\delta}^{a_k})-\sigma^+(a_k)+\sigma^-(a_k)$$
$$-u(\boldsymbol{\delta}^{a_{k+1}})+\sigma^+(a_{k+1})-\sigma^-(a_{k+1})$$

Step 3:

Solve the linear program (7-21) by using formulae (7-33) and (7-34).

Step 4:

Test the existence of multiple or near optimal solutions.

Of course, the ideas employed in all variants of the UTA method are also applicable in the same way in the case of the stochastic UTA.

3.4 UTA-type sorting methods

The extension of the UTA method in the case of a discriminant analysis model was firstly discussed by Jacquet-Lagrèze and Siskos (1982). The aim is to infer u from assignment examples in the context of problematic β (cf. Roy, 1985). In the presence of two classes, if the model is without errors, the following inequalities must hold:

$$\begin{cases} a\in A_1 \Leftrightarrow u[\mathbf{g}(a)]\geq u_o \\ a\in A_2 \Leftrightarrow u[\mathbf{g}(a)]< u_o \end{cases}$$

with u_o being the level of acceptance/rejection which must be found in order to distinguish the set of accepted actions called A_1 and the set of rejected actions called A_2.

Introducing the error variables $\sigma(a)$, $a\in A_R$, the objective is to minimize the sum of deviations from the threshold u_o for the ill classified actions (see *Figure 7-9*). Hence, $u(\mathbf{g})$ can be estimated by means of the LP:

$$\begin{cases} [\min]F = \sum_{a \in A_R} \sigma(a) \\ \text{subject to} \\ \sum_{i=1}^{n} u_i\left[g_i(a)\right] - u_0 + \sigma(a) \geq 0 & \forall a \in A_1 \\ \sum_{i=1}^{n} u_i\left[g_i(a)\right] - u_0 - \sigma(a) \leq 0 & \forall a \in A_2 \\ u_i(g_i^{j+1}) - u_i(g_i^j) \geq s_i & \forall i \text{ and } j \\ \sum_{i=1}^{n} u_i(g_i^*) = 1 \\ u_i(g_{i*}) = 0,\ u_0 \geq 0,\ u_i(g_i^j) \geq 0,\ \sigma(a) \geq 0 & \forall a \in A_R,\ \forall i \text{ and } j \end{cases}$$

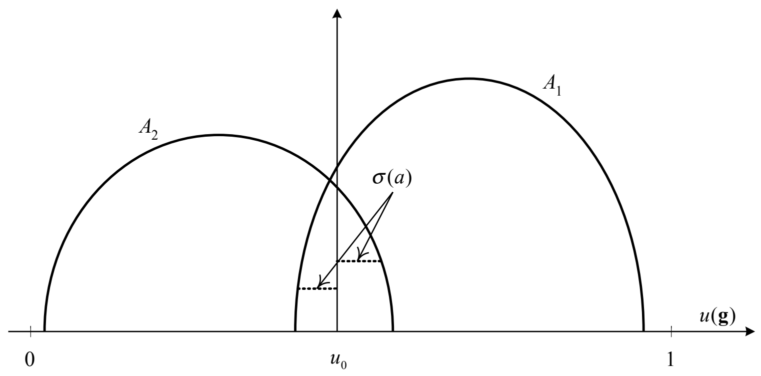

Figure 7-9. Distribution of the actions A_1 and A_2 on $u(\mathbf{g})$ (Jacquet-Lagrèze and Siskos, 1982)

In the general case, the DM's evaluation is expressed in terms of a classification of the reference alternatives into homogenous ordinal groups $A_1 \succ A_2 \succ \ldots \succ A_q$ (i.e. group A_1 includes the most preferred alternatives, whereas group A_q includes the least preferred ones). Within this context, the assessed additive value model will be consistent with the DM's global judgment, if the following conditions are satisfied:

$$\begin{cases} u\left[\mathbf{g}(a)\right] \geq u_1 & \forall a \in A_1 \\ u_l \leq u\left[\mathbf{g}(a)\right] < u_{l-1} & \forall a \in A_l \ (l = 2, 3, \ldots, q-1) \\ u\left[\mathbf{g}(a)\right] < u_{q-1} & \forall a \in A_q \end{cases}$$

where $u_1 > u_2 > \ldots > u_{q-1}$ are thresholds defined in the global value scale $[0,1]$ to discriminate the groups, and u_l is the lower bound of group A_l.

This approach is named UTADIS method (UTilités Additives DIScriminantes) and is presented by Devaud *et al*. (1980), Jacquet-Lagrèze (1995), Zopounidis and Doumpos (1997), Zopounidis and Doumpos (2001), Doumpos and Zopounidis (2002). Similarly, to the UTASTAR method, two error variables are employed in the UTADIS method to measure the differences between the model's results and the predefined classification of the reference alternatives. The additive value model is developed to minimize these errors using a linear programming formulation of type (7-36). In this case, the two types of errors are defined as follows:

1. $\sigma_k^+ = \max\{0, u_l - u[\mathbf{g}(a_k)]\} \quad \forall a_k \in A_l \quad (l = 1,2,\ldots q-1)$ represents the error associated with the violation of the lower bound u_l of a group A_l by an alternative $a_k \in A_l$,
2. $\sigma_k^- = \max\{0, u[\mathbf{g}(a_k)] - u_{l-1}\} \quad \forall a_k \in A_l \quad (l = 2,3,\ldots q)$ represents the error associated with the violation of the upper bound u_{l-1} of a group A_l by an alternative $a_k \in A_l$.

Recently, several new variants of the original UTADIS method have been proposed (UTADIS I, II, III) to consider different optimality criteria during the development of the additive value classification model (Zopounidis and Doumpos, 1997; Zopounidis and Doumpos, 2001; Doumpos and Zopounidis, 2002). The UTADIS I method considers both the minimization of the classification errors, as well as the maximization of the distances of the correctly classified alternatives from the value thresholds. The objective in the UTADIS II method is to minimize the number of misclassified alternatives, whereas UTADIS III combines the minimization of the misclassified alternatives with the maximization of the distances of the correctly classified alternatives from the value thresholds.

In the same context, Zopounidis and Doumpos (2000a) proposed the MHDIS method (Multi-group Hierarchical DIScrimination) extending the preference disaggregation analysis framework of the UTADIS method in complex sorting/classification problems involving multiple-groups. MHDIS addresses sorting problems through a hierarchical (sequential) procedure starting by discriminating group A_1 from all the other groups $\{A_2, A_3, \ldots, A_q\}$, and then proceeding to the discrimination between the alternatives belonging to the other groups. At each stage of this sequential/hierarchical process, two additive value functions are developed for the classification of the alternatives. Assuming that the classification of the alternatives should be made into q ordered classes $A_1 \hbar A_2 \hbar \ldots \hbar A_q$, $2(q-1)$ additive value functions are developed. These value functions have the following additive form:

$$\begin{cases} u_l(\mathbf{g}) = \sum_{i=1}^{n} u_{li}(g_i) \\ u_{:\,l}(\mathbf{g}) = \sum_{i=1}^{n} u_{:\,li}(g_i) \end{cases}$$

where u_l measures the value for the DM of a decision to assign an alternative into group A_l, whereas the $u_{:\,l}$ corresponds to the classification into the set of groups $A_{:\,l} = \{A_{l+1}, A_{l+2}, \ldots, A_q\}$ and both functions are normalized in the interval $[0,1]$.

The rules used to perform the classification of the alternatives have the following form:

$$\begin{cases} \text{if } u_1(a_k) > u_{:\,1}(a_k) \text{ then } a_k \in A_1 \\ \quad \text{else if } u_2(a_k) > u_{:\,2}(a_k) \text{ then } a_k \in A_2 \\ \quad\quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \quad\quad\quad \text{else if } u_{q-1}(a_k) > u_{:\,(q-1)}(a_k) \text{ then } a_k \in A_{q-1} \\ \quad\quad\quad\quad \text{else } a_k \in A_q \end{cases}$$

The development of all value functions in the MHDIS method is performed through the solution of three mathematical programming problems at each stage l of the discrimination process $l = 1, 2, \ldots q-1$. Initially, a LP is solved to minimize the magnitude of the classification errors (in distance terms similarly to the UTADIS approach). Then, a mixed-integer LP is solved to minimize the total number of misclassifications among the misclassifications that occur after the solution of the initial LP, while retaining the correct classifications. Finally, a second LP is solved to maximize the clarity of the classification obtained from the solutions of the previous LPs.

3.5 Other variants and extensions

In all previous approaches, the value function was built in a one-step process by formulating a LP that requires only the DM's global preferences. In some cases, however, it would be more appropriate to build such a function from a two-step questioning process, by dissociating the construction of the marginal value functions and the assessment of their respective scaling constants.

In the first step, the various marginal value functions are built outside the UTA algorithm. These functions may be facilitated, for instance, by proposing specific parametrical marginal value functions to the DM and asking him/her to choose the one that matches his/her preferences on that specific criterion. Those functions should be normalized according to (7-4) conditions. Generally, the approaches applied in this construction step are:

a) techniques based on MAUT theory and described by Keeney and Raiffa (1976), and Klein *et al*. (1985),
b) the MACBETH method (Bana e Costa and Vansnick, 1994, 1997; Bana e Costa *et al*., 2001),
c) the Quasi-UTA method by Beuthe *et al*. (2000), that uses "recursive exponential" marginal value functions, and
d) the MIIDAS system (see section 4) that combines artificial intelligence and visual procedures in order to extract the DM's preferences (Siskos *et al*., 1999).

In the second step, after the assessment of these value functions, the DM is asked to give a global ranking of alternatives in a similar way as in the basic UTA method. From this information, the problem may be formulated via a LP, in order to assess only the weighting factors p_i of the criteria (scaling constants of criteria). Through this approach, initially named UTA II model (Siskos, 1980), formula (7-11) becomes:

$$\Delta(a,b) = \sum_{i=1}^{n} p_i \{ u_i[g_i(a)] - u_i[g_i(b)] \} - \sigma^+(a) + \sigma^-(a) + \sigma^+(b) - \sigma^-(b)$$

and the LP (7-14) is modified as follows

$$\begin{cases} [\min] F = \sum_{a \in A_R} \left[\sigma^+(a) + \sigma^-(a) \right] \\ \text{subject to} \\ \Delta(a,b) \geq \delta & \text{if } a \,\hbar\, b \\ \Delta(a,b) = 0 & \text{if } a : b \\ \sum_{i=1}^{n} p_i = 1 \\ p_i \geq 0,\ \sigma^+(a) \geq 0,\ \sigma^-(a) \geq 0 \quad \forall a \in A_R,\ \forall i \end{cases}$$

The main principles of the UTA methods are also applicable in the specific field of multiobjective optimization, mainly in the field of linear programming with multiple objective functions. For instance, in the classical methods of Geoffrion *et al*. (1972) and Zionts and Wallenius (1976), the weights of the linear combinations of the objectives are inferred locally from trade–offs or pairwise judgments given by the DM at each iteration of the methods. Thus, these methods exploit in a direct way the DM's value functions and seek the best compromise solution through successive maximization of these assessed value functions.

Stewart (1987) proposed a procedure of pruning the decision alternatives using the UTA method. In this approach a sequence of alternatives is presented to the DM, who places each new presented alternative in rank order relative to the earlier alternatives evaluated. This ranking of elements in a subset of the decision space is used to eliminate other alternatives from further consideration. In the same context, Jacquet-Lagrèze *et al*. (1987) developed a disaggregation method, similar to UTA, to assess a whole value function of multiple objectives for linear programming systems. This methodology enables to find compromise solutions and is mainly based on the following steps:

1. Generation of a limited subset of feasible efficient solutions as representative as possible of the efficient set.
2. Assessment of an additive value function using PREFCALC system (see section 4).
3. Optimization of the additive value function on the original set of feasible alternatives.

Also, Siskos and Despotis (1989), in the context of UTA-based approaches in multiobjective optimization problems, proposed the ADELAIS method. This approach refers to an interactive method that uses UTA iteratively, in order to optimize an additive value function within the feasible region defined on the basis of the satisfaction levels and determined in each iteration.

3.6 Other disaggregation methods

The main principles of the aggregation-disaggregation approach may be combined with outranking relation methods. The most important efforts concern the problem of determining the values of several parameters when using these methods. The set of these parameters is used to construct a preference model with which the DM accepts as a working hypothesis in the decision aid study. Assuming that the DM is able to give explicitly the values of each parameter is not realistic in several real-world applications.

In this framework, the ELECCALC system has been developed (Kiss *et al*., 1994), which estimates indirectly the parameters of the ELECTRE II method. The process is based on the DM's responses to questions of the system regarding his/her global preferences.

Furthermore, concerning problematic β, several approaches consist in inferring the parameters of ELECTRE TRI through holistic information on DM's judgments. These approaches aim at substituting assignment examples for direct elicitation of the model parameters. Usually, the values of these parameters are inferred through a regression-type analysis on assignment examples.

Mousseau and Slowinski (1998) propose an interactive aggregation-disaggregation approach that infers ELECTRE TRI parameters simultaneously starting from assignment examples. In this approach, the determination of the parameters' values (except the veto thresholds) that best restore the assignment examples is formulated through a nonlinear optimization program.

Several efforts have tried to overcome the limitations of the aforementioned approach (computational difficulty, estimation of the veto threshold):

a) Mousseau *et al*. (2000a; 2000b) consider the subproblem of the determination of the weights only, assuming that the thresholds and category limits have been fixed. This leads to formulate a linear program (rather than nonlinear in the global inference model). Through experimental analysis, they show that this approach is able to infer weights that restore in a stable way the assignment examples and it is also able to identify possible inconsistencies in these assignment examples.
b) Doumpos and Zopounidis (2002) use linear programming formulations in order to estimate all the parameters of the outranking relation classification model. However, in this approach, the parameters are estimated sequentially rather than through a global inference process. Thus, the proposed methodology does not specify the optimal parameters of the outranking relation (i.e. the ones that lead to a global minimum of the classification error). Therefore, the results of this approach ("reasonable" specification of the parameters) serve rather as a basis for a thorough decision-aid process.

The problem of robustness and sensitivity analysis, through the extension of the previous research efforts is discussed by Dias *et al*. (2002). They consider the case where the DM can not provide exact values for the parameters of the ELECTRE TRI method, due to uncertain, imprecise or inaccurately determined information, as well as from lack of consensus among them. The proposed methodology combines the following approaches:

1. The first approach infers the value of parameters from assignment examples provided by the DM, as an elicitation aid.
2. The second approach considers a set of constraints on the parameter values reflecting the imprecise information that the DM is able to provide.

In the context of ordinal regression analysis, the MUSA method has been developed in order to measure and analyze customer satisfaction (Siskos *et al*., 1998; Grigoroudis and Siskos, 2002). The method is used for the assessment of a set of marginal satisfaction functions in such a way that the global satisfaction criterion becomes as consistent as possible with customer's judgments. Thus, the main objective of the method is the aggregation of individual judgments into a collective value function.

The MUSA method assesses global and partial satisfaction functions Y^* and X_i^* respectively, given customers' ordinal judgments Y and X_i (for the i-th criterion). The ordinal regression analysis equation has the following form:

$$\tilde{Y}^* = \sum_{i=1}^{n} b_i X_i^* - \sigma^+ + \sigma^-$$

where $\tilde{Y}^*$ is the estimation of the global value function Y^*, n is the number of criteria, b_i is a positive weight of the i-th criterion, σ^+ and σ^- are the overestimation and the underestimation errors, respectively, and the value functions Y^* and X_i^* are normalized in the interval [0,100].

Similarly to the UTASTAR algorithm, the following transformation equations are used:

$$\begin{cases} z_m = y^{*m+1} - y^{*m} & \text{for } m = 1,2,...,\alpha - 1 \\ w_{ik} = b_i x_i^{*k+1} - b_i x_i^{*k} & \text{for } k = 1,2,...,\alpha_i - 1 \text{ and } i = 1,2,...,n \end{cases}$$

where y^{*m} is the value of the y^m satisfaction level, x_i^{*k} is the value of the x_i^k satisfaction level, and α and α_i are the number global and partial satisfaction levels.

According to the previous definitions and assumptions, the MUSA estimation model can be written in a LP formulation, as follows:

$$\begin{cases} [\min] F = \sum_{j=1}^{M} \sigma_j^+ + \sigma_j^- \\ \text{subject to} \\ \sum_{i=1}^{n} \sum_{k=1}^{x_i^j - 1} w_{ik} - \sum_{m=1}^{y^j - 1} z_m - \sigma_j^+ + \sigma_j^- = 0 \quad \text{for } j = 1,2,\dots,M \\ \sum_{m=1}^{\alpha - 1} z_m = 100 \\ \sum_{i=1}^{n} \sum_{k=1}^{\alpha_i - 1} w_{ik} = 100 \\ z_m,\ w_{ik},\ \sigma_j^+,\ \sigma_j^- \quad \forall\, m,i,j,k \end{cases}$$

where M is the size of the customer sample, and x_i^j and y^j are the j-th level on which variables X_i and Y are estimated (i.e. global and partial satisfaction judgments of the j-th customer). The MUSA method includes also a post-optimality analysis stage, similarly to step 4 of the UTASTAR algorithm.

An analytical development of the method and the provided results is given by Grigoroudis and Siskos (2002), while the presentation of the

MUSA DSS can be found in Grigoroudis *et al*. (2000) and Grigoroudis and Siskos (2003).

The problem of building non-additive utility functions may also be considered in the context of aggregation-disaggregation approach. A characteristic case refers to positive interaction (synergy) or negative interaction among criteria (redundancy). Two or more criteria are synergic (redundant) when their joint weight is more (less) than the sum of the weights given to the criteria considered singularly.

In order to represent interaction among criteria, some specific formulations of the utility functions expressed in terms of fuzzy integrals have been proposed (Murofushi and Sugeno, 1989; Grabisch, 1996; Marichal and Roubens, 2000). In this context, Angilella *et al*. (2003) propose a methodology that allows including additional information such as an interaction among criteria. The method aims at searching a utility function representing the DM's preferences, while the resulting functional form is a specific fuzzy integral (Choquet integral). As a result, the obtained weights may be interpreted as the "importance" of coalitions of criteria, exploiting the potential interaction between criteria. The method can also provide the marginal utility functions relative to each one of the considered criteria, evaluated on a common scale, as a consequence of the implemented methodology.

The general scheme of the disaggregation philosophy is also employed in other approaches, including rough sets (Pawlak, 1982; Slowinski, 1995; Dimitras, *et al*., 1999; Zaras, 2000), machine learning (Quinlan, 1986) and neural networks (Malakooti and Zhou, 1994; Stam *et al*., 1996). All these approaches are used to infer some form of decision model (a set of decision rules or a network) from given decision results involving assignment examples, ordinal or measurable judgments.

4. APPLICATIONS AND UTA-BASED DSS

The methods presented in the previous sections adopt the aggregation-disaggregation approach. This approach constitutes a basis for the interaction between the analyst and the DM, which includes:

- the consistency between the assessed preference model and the a priori preferences of the DM,
- the assessed values (values, weights, utilities, …), and
- the overall evaluation of potential actions (extrapolation output).

A general interaction scheme for this decision support process is given in *Figure 7-10*.

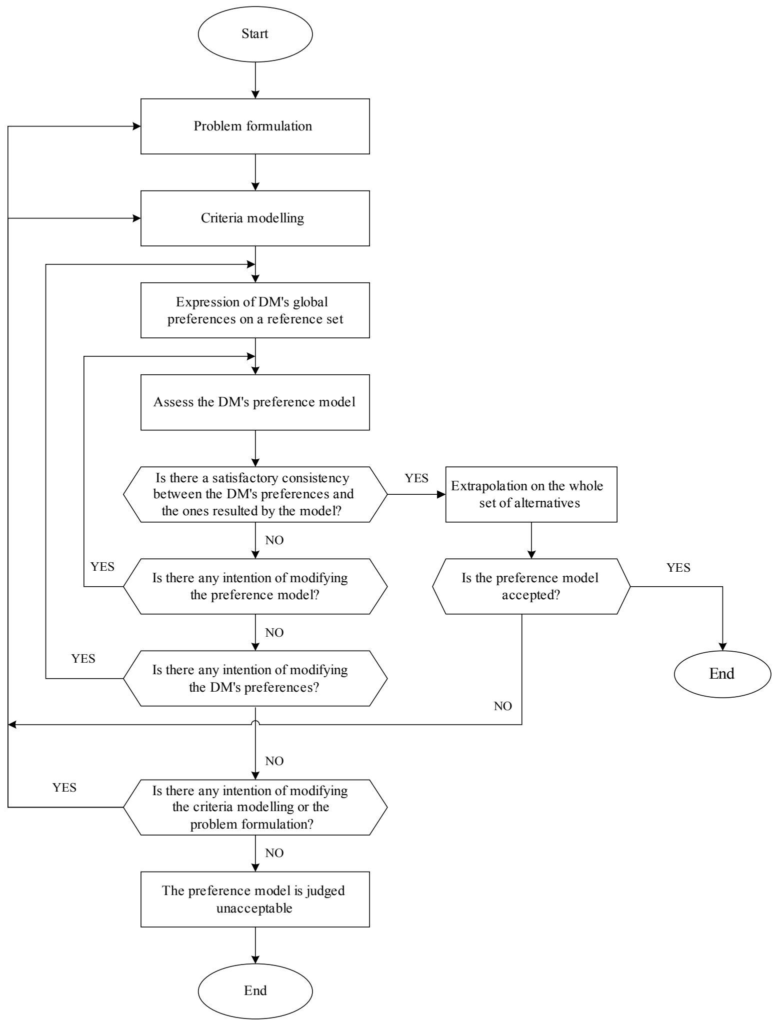

Figure 7-10. Simplified decision support process based on disaggregation approach (Jacquet-Lagrèze and Siskos, 2001)

Several decision support systems (DSSs), based on the UTA model and its variants, have been developed on the basis of disaggregation methods. These systems include:

a) The PREFCALC system (Jacquet-Lagrèze, 1990) is a DSS for interactive assessment of preferences using holistic judgments. The interactive process includes the classical aggregation phase where the DM is asked to estimate directly the parameters of the model (i.e. weights, trade-offs, etc.), as well as the disaggregation phase where the DM is asked to express his/her holistic judgments (i.e. global preference order on a subset of the alternatives) enabling an indirect estimation of the parameters of the model.
b) MINORA (Multicriteria Interactive Ordinal Regression Analysis) is a multicriteria interactive DSS with a wide spectrum of supported decision making situations (Siskos *et al*., 1993, 1994). The core of the system is based on the UTASTAR method and it uses special interaction techniques in order to guide the DM to reach a consistent preference system.
c) MIIDAS (Multicriteria Interactive Intelligence Decision Aiding System) is an interactive DSS that implements the extended UTA II method (Siskos *et al*., 1999). In the first step of the decision-aiding process, the system assess the DM's value functions, while in the next step, the system estimates the DM's preference model from his/her global preferences on a reference set of alternative actions. The system uses Artificial Intelligence and Visual techniques in order to improve the user interface and the interactive process with the DM (*Figure 7-11*).
d) The UTA PLUS software (Kostkowski and Slowinski, 1996; http://www.lamsade.dauphine.fr/english/software.html#uta+) is an implementation of the UTA method, which allows the user to modify interactively the marginal value functions within limits following from a sensitivity analysis of the formulated ordinal regression problem. During all these modifications, a friendly graphical interface helps the DM to reach an accepted preference model.
e) MUSTARD (Multicriteria Utility-based Stochastic Aid for Ranking Decisions) is an interactive DSS developed by Beuthe and Scannella (1999), which incorporates several variants of the UTA method. The system provides several visual tools in order to structure the DM's preferences to a specific problem (see also Siskos, 2002). The interactive process with the DM contains the following main steps: problem structuring, preference questionnaire, optimization solver-parameter computing, final results (full rankings and graphs).

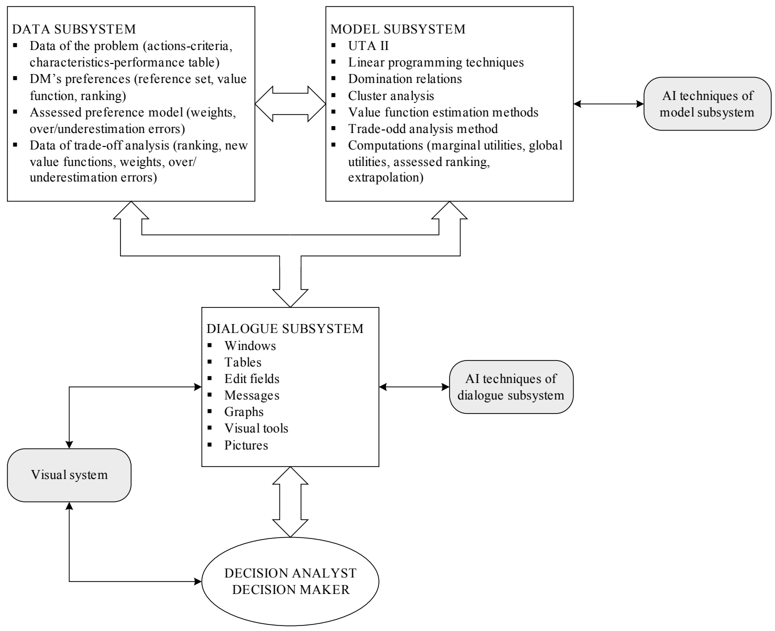

Figure 7-11. Components of MIIDAS system (Siskos, *et al*., 1999)

UTA methods have also been used in several works for conflict resolution in multi-actor decision situations (Jacquet-Lagrèze and Shakun, 1984; Bui, 1987; Matsatsinis and Samaras, 2001). In the same context, the MEDIATOR system was developed (Jarke *et al*., 1988; Shakun, 1988; Shakun, 1991), which is a negotiation support system based on Evolutionary Systems Design (ESD) and database-centered implementation. ESD visualizes negotiations as a collective process of searching for designing a mutually acceptable solution. Participants are seen as playing a dynamical difference game in which a coalition of players is formed, if it can achieve a set of agreed upon goals. In MEDIATOR, negotiations are supported by consensus seeking through exchange of information and, where consensus is incomplete, by compromise. It assists in consensus seeking by aiding the players to build a group joint problem representation of the negotiations-in effect, joint mappings from control space to goal space (and through marginal utility functions) to utility space. Individual marginal utility functions are estimated by applying the UTA method. Players can arrive to a common coalition utility function through exchange of information and negotiation until players' marginal utility functions are identical. In addition to exchanging information and negotiating to expand targets, players can consider the use of axioms to contract the feasible region.

In the area of intelligent multicriteria DSSs, the MARKEX system has been proposed by Siskos and Matsatsinis (1993), Matsatsinis and Siskos (1999), Matsatsinis and Siskos (2003). The system includes the UTASTAR algorithm and is used for the new product development process. It acts as a consultant for marketers, providing visual support to enhance understanding and to overcome lack of expertise. The data bases of the system are the results of consumer surveys, as well as financial information of the enterprises involved in the decision making process. The system's model base encompasses statistical analysis, preference analysis, and consumer choice models. *Figure 7-12* presents a general methodological flowchart of the system. Also, MARKEX incorporates partial knowledge bases to support decision makers in different stages of the product development process. The system incorporates three partial expert systems, functioning independently of each other: They use the following knowledge bases for the:

- selection of data analysis method,
- selection of brand choice model, and
- evaluation of the financial status of enterprises.

Furthermore, an intelligent web-based DSS, named DIMITRA, has been developed by Matsatsinis and Siskos (2001). The system is a consumer survey-based DSS, focusing on the decision-aid process for agricultural product development. Besides the implementation of the UTASTAR method in the preference analysis module, the DIMITRA system comprises several statistical analysis tools and consumer choice models. The system provides visual support to the DM (agricultural cooperatives, agribusiness firms, etc.) for several complex tasks, such as:

- evaluation of current and potential market shares,
- determination of the appropriate communication and penetration strategies, based on consumer attitudes and beliefs,
- adjustment of the production according to product's demand, and
- detection of the most promising markets.

In the same context, new research efforts have combined UTA-based DSSs with intelligent agents' technology. In general, the proposed methodologies engage the UTA models in a multi-agent architecture in order to assess the DM's preference system. These research efforts include mainly the following:

a) An intelligent agent-based DSS, focusing on the determination of product penetration strategies has been developed by Matsatsinis *et al.* (1999, 2000, 2001). The system implements an original consumer-based methodology, in which intelligent agents operate in a functional and a structural level, simultaneously. Task, information and interface agents are included in the functional level in order to coordinate, collect necessary information and communicate with the DM. Likewise, the structural level includes elementary agents based on a generic reusable architecture and complex agents which aim to the development of a dynamical agent organization in a recursive way.

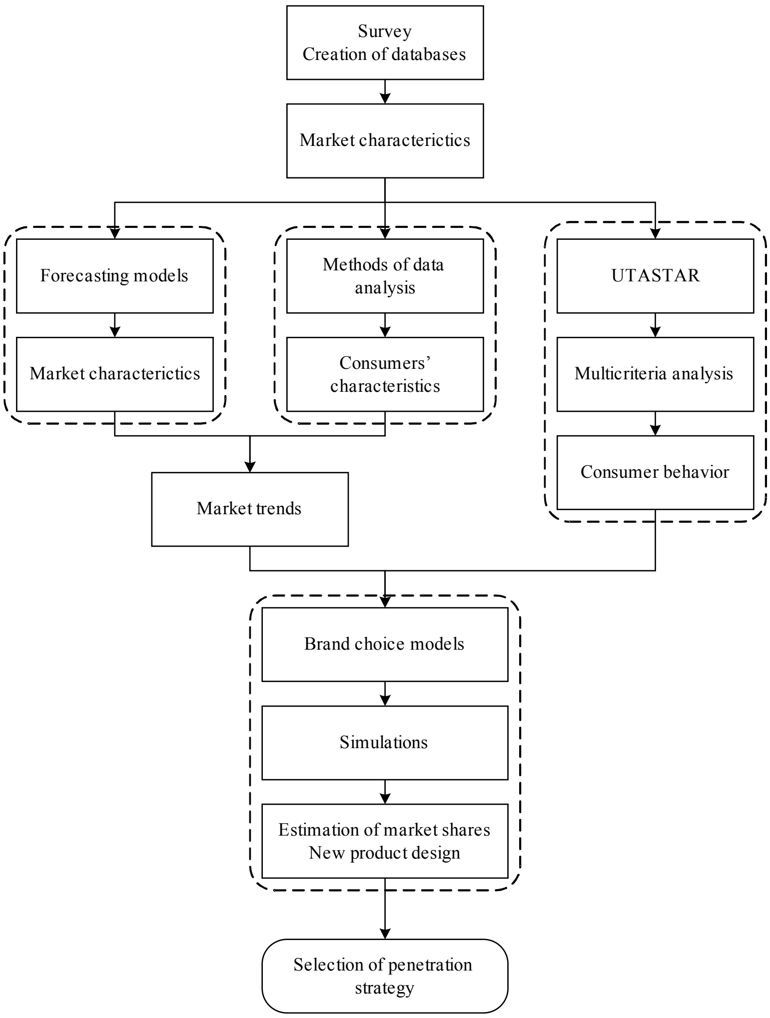

Figure 7-12. Methodological flowchart of MARKEX (Matsatsinis and Siskos, 1999)

b) A multi-agent architecture is proposed by Manouselis and Matsatsinis (2001) for modeling electronic consumer's behavior. The implementation of the system refers to electronic marketplaces and incorporates a step-by-step methodology for intelligent systems analysis and design, used in the particular decision-aid process. The system develops consumer behavioral models for the purchasing and negotiation process adopting additional operations research tools and techniques. The presented application refers to the case of Internet radio.
c) The AgentAllocator system (Matsatsinis and Delias, 2003) implements the UTA II method in the task allocation problem. These problems are very common to any multi-agent system in the context of Artificial Intelligence. The system is an intelligent agent DSS, which allows the DM to model his/her preferences in order to reach and employ the optimal allocation plan.

The need to combine data and knowledge in order to solve complex and ill-structured decision problems is a major concern in the modern marketing-management science. Matsatsinis (2002) has proposed a DSS that implements the UTASTAR algorithm along with rule-induction data mining techniques. The main aim of the system is to derive and apply a set of rules that relate the global and the marginal value functions. A comparison between the original and the rule-based global values is used in the validity and stability analysis of the proposed methodology.

Furthermore, in the area of financial management, a variety of UTA-based DSSs has been developed, including mainly the following systems:

a) The FINEVA system (Zopounidis *et al.*, 1996) is a multicriteria knowledge-based DSS developed for the assessment of corporate performance and viability. The system implements multivariate statistical techniques (e.g. principal components analysis), expert systems technology, and the UTASTAR method to provide integrated support in evaluating the corporate performance.
b) The FINCLAS system (Zopounidis and Doumpos, 1998) is a multicriteria DSS developed to study financial decision-making problems in which a classification (sorting) of the alternatives is required. The present form of the system is devoted to corporate credit risk assessment, and it can be used to develop classification models to assign a set of firms into predefined credit risk classes. The analysis performed by the system is based on the family of the UTADIS methods.
c) The INVESTOR system (Zopounidis and Doumpos, 2000b) is developed to study problems related to portfolio selection and management. The system implements the UTADIS method, as well as

goal programming techniques to support portfolio managers and investors in their daily practice.

d) The PREFDIS system (Zopounidis and Doumpos, 2000c) is a multicriteria DSS developed to address classification problems. The system implements a series of preference disaggregation analysis techniques, namely the family of the UTADIS methods, in order to develop an additive utility function to be used for classification purposes

Finally, as presented in section 3.5, Siskos and Despotis (1989) have developed the ADELAIS system which is designed to decision-aid in multiobjective linear programming (MOLP) problems.

Over the past two decades UTA-based methods have been applied in several real-world decision-making problems from the fields of financial management, marketing, environmental management, as well as human resources management, as presented in *Table 7-1*. These applications have provided insight on the applicability of preference disaggregation analysis in addressing real-world decision problems and its efficiency.

Table 7-8. Indicative applications of the UTA methods

Field	Scope	Reference
Financial management	Venture capital evaluation	Siskos and Zopounidis (1987)
	Portfolio selection and management	Hurson and Zopounidis (1997)
		Zopounidis *et al*. (1999)
	Business failure prediction	Zopounidis (1987)
		Zopounidis and Doumpos (1999)
	Business financing	Siskos *et al*. (1994)
		Zopounidis *et al*. (1996)
		Zopounidis and Doumpos (1998)
	Country risk assessment	Cosset *et al*. (1992)
		Oral *et al*. (1992)
		Zopounidis *et al*. (2000)
Marketing	Marketing of new products	Spiliopoulos (1987)
	Marketing of agricultural products	Baourakis *et al*. (1993)
		Siskos and Matsatsinis (1993)
		Baourakis *et al*. (1996)
		Matsatsinis *et al*. (1999, 2000)
		Siskos *et al*. (2001)
		Matsatsinis and Siskos (2001)
		Matsatsinis and Siskos (2003)
	Consumer behavior	Siskos *et al*. (1995a)
		Siskos *et al*. (1995b)
		Baourakis *et al*. (1995)
		Manouselis and Matsatsinis (2001)
		Matsatsinis (2002)
	Customer satisfaction	Grigoroudis *et al*. (1999)
		Mihelis *et al*. (2001)
		Siskos *et al*. (2001)

Field	Scope	Reference
		Siskos and Grigoroudis (2002)
		Grigoroudis *et al*. (2002)
		Grigoroudis and Siskos (2003)
		Sandalidou *et al*. (2003)
	Sales strategy problems	Richard (1983)
		Siskos (1986)
Management (general)	Project evaluation	Jacquet-Lagrèze (1995)
		Beuthe *et al*. (2000)
	Environmental management	Siskos and Assimakopoulos (1989)
		Hatzinakos *et al*. (1991)
		Diakoulaki *et al*. (1999)
	Job evaluation	Spyridakos *et al*. (2000)
		González-Araya *et al*. (2002)

5. CONCLUDING REMARKS AND FUTURE RESEARCH

The UTA methods presented in this chapter belong to the family of ordinal regression analysis models aiming to assess a value system as a model of the preferences of the DM. This assessment is implemented through an aggregation-disaggregation process. With this process the analyst is able to infer an analytical model of preferences, which is as consistent as possible with the DM' preferences. The acceptance of such a preference model is accomplished through a repetitive interaction between the model and the DM. This approach contributes towards an alternative reasoning for decision-aid.

Future research regarding UTA methods aims to explore further the potentials of the preference disaggregation philosophy within the context of multicriteria decision-aid. Jacquet-Lagrèze and Siskos (2001) propose that potential research developments may be focused on:

a) the inference of more sophisticated aggregation models by disaggregation, and
b) the experimental evaluation of disaggregation procedures.

Finally, it would be interesting to explore the relationship of aggregation and disaggregation procedures in terms of similarities and/or dissimilarities regarding the evaluation results obtained by both approaches (Jacquet-Lagrèze and Siskos, 2001). This will enable the identification of the reasons and the conditions under which aggregation and disaggregation procedures will lead to different or the same results.

REFERENCES

Angilella, S., S. Greco, F. Lamantia and B. Matarazzo (2003). Assessing non-additive utility for multicriteria decision aid, *European Journal of Operational Research* (to appear).

Bana e Costa, C.A. and J.C. Vansnick (1994). MACBETH: an interactive path towards the construction of cardinal value functions, *International Transactions in Operations Research*, 1 (4), 489-500.

Bana e Costa, C.A. and J.C. Vansnick (1997). Applications of the MACBETH approach in the framework of an additive aggregation model, *Journal of Multi-Criteria Decision Analysis*, 6 (2), 107-114.

Bana e Costa, C.A., F. Nunes Da Silva and J. C. Vansnick (2001). Conflict dissolution in the public sector: A case-study, *European Journal of Operational Research*, 130 (2), 388-401.

Baourakis, G., N.F. Matsatsinis and Y. Siskos (1993). Agricultural product design and development, in: J. Janssen and C.H. Skiadas, (eds.), *Applied stochastic models and data analysis*, World Scientific, 1108-1128.

Baourakis, G., N.F. Matsatsinis and Y. Siskos (1995). Consumer behavioural analysis using multicriteria methods, in: J. Janssen, C.H. Skiadas and C. Zopounidis, (eds.), *Advances in stochastic modelling and data analysis*, Kluwer Academic Publishers, Dordrecht, 328-338.

Baourakis, G., N.F. Matsatsinis and Y. Siskos (1996). Agricultural product development using multidimensional and multicriteria analyses: The case of wine, *European Journal of Operational Research*, 94 (2), 321-334.

Beuthe, M. and G. Scannella (1996). Applications comparées des méthodes d'analyse multicritère UTA, *RAIRO Recherche Opérationelle*, 30 (3), 293-315.

Beuthe, M. and G. Scannella (1999). *MUSTARD user's guide*, GTM, Facultés Universitaires Catholiques de Mons (FUCaM), Mons.

Beuthe, M. and G. Scannella (2001). Comparative analysis of UTA multicriteria methods, *European Journal of Operational Research*, 130 (2), 246–262.

Beuthe, M., L. Eeckhoudt and G. Scanella (2000). A practical multicriteria methodology for assessing risky public investments, *Socio-Economic Planning Sciences*, 34 (2), 121-139.

Bui, T.X. (1987). *Co-oP: A group decision support system for cooperative multiple criteria group decision making*, Lecture Notes in Computer Science, No. 290, Springer-Verlag, Berlin.

Charnes, A. and W. Cooper (1961). *Management models and industrial applications of linear programming Vol. 1*, Wiley, New York.

Charnes, A., W. Cooper, and R.O. Ferguson (1955). Optimal estimation of executive compensation by linear programming, *Management Science*, 1 (2), 138-151.

Cosset, J.C., Y. Siskos and C. Zopounidis (1992). Evaluating country risk: A decision support approach, *Global Finance Journal*, 3 (1), 79-95.

Despotis, D.K. and C. Zopounidis (1993). Building additive utilities in the presence of non-monotonic preference, in: P.M. Pardalos, Y. Siskos and C. Zopounidis (eds.), *Advances in multicriteria analysis*, Kluwer Academic Publisher, Dordrecht, 101-114.

Despotis, D.K. and D. Yannacopoulos (1990). Méthode d'estimation d'utilités additives concaves en programmation linéare multiobjectifs, *RAIRO Recherche Opérationelle*, 24, 331-349.

Despotis, D.K., D. Yannacopoulos and C. Zopounidis (1990). A review of the UTA multicriteria method and some improvements, *Foundations of Computing and Decision Sciences*, 15 (2), 63–76.

Devaud, J.M., G. Groussaud and E. Jacquet-Lagrèze (1980). UTADIS: Une méthode de construction de fonctions d'utilité additives rendant compte de jugements globaux, *European Working Group on Multicriteria Decision Aid*, Bochum.

Diakoulaki, D., C. Zopounidis, G. Mavrotas and M. Doumpos (1999). The use of a preference disaggregation method in energy analysis and policy making", *Energy–The International Journal*, 24 (2), 157-166.

Dias, L., V. Mousseau, J. Figueira and J. Climaco (2002). An aggregation/disaggregation approach to obtain robust conclusions with ELECTRE TRI, *European Journal of Operational Research*, 138 (2), 332-348.

Dimitras, A.I., R. Slowinski, R. Susmaga, and C. Zopounidis (1999). Business failure prediction using rough sets, *European Journal of Operational Research*, 114 (2), 263-280.

Doumpos, M. and C. Zopounidis (2002). *Multicriteria decision aid classification methods*, Kluwer Academic Publishers, Dordrecht.

Doumpos, M. and C. Zopounidis (2002). On the development of an outranking relation for ordinal classification problems: An experimental investigation of a new methodology, *Optimization Methods and Software*, 17 (2), 293-317.

Fishburn, P. (1966). A note on recent developments in additive utility theories for multiple factors situations, *Operations Research*, 14, 1143-1148.

Fishburn, P. (1967). Methods for estimating additive utilities, *Management Science*, 13, 435-453.

Freed, N. and G. Glover (1981). Simple but powerful goal programming models for discriminant problems, *European Journal of Operational Research*, 7, 44-60.

Geoffrion, A.M., J.S. Dyer, and A. Feinberg (1972). An interactive approach for multi-criterion optimization, with an application to the operation of an academic department, *Management Science*, 19 (4), 357-368.

González-Araya, M.C., L.A.D. Rangel, M.P.E. Lins and L.F.A.M. Gomes (2002). Building the additive utility functions for CAD-UFRJ evaluation staff criteria, *Annals of Operations Research*, 116 (1-4), 271-288.

Grabisch, M. (1996). The application of fuzzy integrals in multicriteria decision making, *European Journal of Operational Research*, 89 (3), 445-456.

Grigoroudis, E. and Y. Siskos (2002). Preference disaggregation for measuring and analysing customer satisfaction: The MUSA method, European Journal of Operational Research, 143 (1), 148-170.

Grigoroudis, E. and Y. Siskos (2003). A survey of customer satisfaction barometers: Results from the transportation-communications sector, *European Journal of Operational Research*, 152 (2), 334-353.

Grigoroudis, E. and Y. Siskos (2003). MUSA: A decision support system for evaluating and analysing customer satisfaction, in K. Margaritis and I. Pitas (eds.), *Proceedings of the 9th Panhellenic Conference in Informatics*, Thessaloniki, Greece, 113-127.

Grigoroudis, E., J. Malandrakis, J. Politis and Y. Siskos (1999). Customer satisfaction measurement: An application to the Greek shipping sector, *Proceedings of the 5th Decision Sciences Institute's International Conference on Integrating Technology & Human Decisions: Global Bridges into the 21st Century*, Athens, Greece, 2, 1363-1365.

Grigoroudis, E., Siskos Y. and O. Saurais (2000). TELOS: A customer satisfaction evaluation software, *Computers and Operations Research*, 27 (7-8), 799-817.

Grigoroudis, E., Y. Politis and Y. Siskos (2002). Satisfaction benchmarking and customer classification: An application to the branches of a banking organization, *International Transactions in Operational Research*, 9 (5), 599-618.

Hatzinakos, I., D. Yannacopoulos, C. Faltsetas and C. Ziourkas (1991). Application of the MINORA decision support system to the evaluation of landslide favourability in Greece, *European Journal of Operational Research*, 50 (1), 60-75.

Hurson, Ch. and C. Zopounidis (1997). *Gestion de portefeuille et analyse multicritère*, Economica, Paris.

Jacquet-Lagrèze, E. (1990). Interactive assessment of preferences using holistic judgments: The PREFCALC system, in: C.A. Bana e Costa (ed.), *Readings in multiple criteria decision aid*, Springer-Verlag, Berlin, 225-250.

Jacquet-Lagrèze, E. (1995). An application of the UTA discriminant model for the evaluation of R&D projects, in: P.M. Pardalos, Y. Siskos and C. Zopounidis (eds.), *Advances in multicriteria analysis*, Kluwer Academic Publishers, Dordrecht, 203–211.

Jacquet-Lagrèze, E. and J. Siskos (1978). Une méthode de construction de fonctions d' utilité additives explicatives d' une préférence globale, *Cahier du LAMSADE*, 16, Université de Paris-Dauphine.

Jacquet-Lagrèze, E. and M.F. Shakun (1984). Decision support systems for semistructured buying decisions, *European Journal of Operational Research*, 16, 48-56.

Jacquet-Lagrèze, E. and Y. Siskos (1982). Assessing a set of additive utility functions for multicriteria decision making: The UTA method, *European Journal of Operational Research*, 10 (2), 151–164.

Jacquet-Lagrèze, E. and Y. Siskos (2001). Preference disaggregation: 20 years of MCDA experience, *European Journal of Operational Research*, 130 (2), 233–245.

Jacquet-Lagrèze, E., R. Meziani and R. Slowinski (1987). MOLP with an interactive assessment of a piecewise linear utility function, *European Journal of Operational Research*, 31 (3), 350-357.

Jarke, M., M.T. Jelassi and M.F. Shakun (1987). MEDIATOR: Toward a negotiation support system, *European Journal of Operational Research*, 31 (3), 314-334.

Karst, O.J. (1958). Linear curve fitting using least deviations, *Journal of the American Statistical Association*, 53, 118-132.

Keeney, R.L. (1980). *Sitting energy facilities*, Academic Press, New York.

Keeney, R.L. and H. Raiffa (1976). *Decisions with multiple objectives: Preferences and value tradeoffs*, John Wiley and Sons, New York.

Kelley, J.E. (1958). An application of linear programming to curve fitting, *Journal of Industrial and Applied Mathematics*, 6 (1), 15-22.

Kiss, L.N., J.M. Martel and R. Nadeau (1994). ELECCALC-An interactive software for modelling the decision maker's preferences, *Decision Support Systems*, 12 (4-5), 311-326.

Klein, G., H. Moskowitz, S. Mahesh and A. Ravindran (1985). Assessment of multiattribute measurable value and utility functions via mathematical programming, *Decision Sciences*, 16 (3), 309-324.

Kostkowski, M. and R. Slowinski (1996). UTA+ Application (v. 1.20)-User's manual, *Document du LAMSADE*, No. 95, Université de Paris-Dauphine, Paris.

Malakooti, B. and Y.Q. Zhou (1994). Feedforward artificial neural networks for solving discrete multiple criteria decision making problems, *Management Science*, 40 (11), 1542-1561.

Manas, M. and J. Nedoma (1968). Finding all vertices of a convex polyhedron, *Numerical Mathematics*, 12, 226-229.

Manouselis, N. and N.F. Matsatsinis (2001). Introducing a multi-agent, multi-criteria methodology for modeling electronic consumer's behavior: The case of internet radio, in: M. Klush and F. Zambonelli (eds.), *Lecture notes in artificial intelligence-Cooperative information agents*, Springer Verlag, 2182, pp. 190-195.

Marichal, J.L. and M. Roubens (2000). Determination of weights of interactive criteria from a reference set, *European Journal of Operational Research*, 124 (3), 641-650.

Matsatsinis, N.F. (2002). New agricultural product development using data mining techniques and multicriteria methods, in: *Proceedings of the 1st Hellenic Association of Information and Communication Technology in Agriculture, Food and Environment (HAICTA's Conference 2002)*, June 6-7, Athens, Greece.

Matsatsinis, N.F. and A. Samaras (2001). MCDA and preference disaggregation in group decision support systems, *European Journal of Operational Research*, 130 (2), 414-429.
Matsatsinis, N.F. and P. Delias (2003). AgentAllocator: An agent-based multi-criteria decision support system for task allocation, in: V. Marik D. McFarlane and P. Valckenaers, *Holonic and multi-agent systems for manufacturing*, *Lecture notes in computer science*, Springer Verlag (to appear).
Matsatsinis, N.F. and Y. Siskos (1999). MARKEX: An intelligent decision support system for product development decision, *European Journal of Operational Research*, 113 (2), 336-354.
Matsatsinis, N.F. and Y. Siskos (2001). DIMITRA: An intelligent decision support system for agricultural products development decisions, in: *Proceedings of the 3rd European Conference of the European Federation for Information Technology in Agriculture, Food and the Environment (EFITA 2001)*, June 18-20, Montpellier, France.
Matsatsinis, N.F. and Y. Siskos (2003). *Intelligent support systems for marketing decision*, Kluwer Academic Publishers, Dordrecht.
Matsatsinis, N.F., P. Moraitis, V. Psomatakis and N. Spanoudakis (1999). Intelligent software agents for products penetration strategy selection, in: *Proceedings of Modelling Autonomous Agents in a Multi-Agent World 1999 (MAAMAW'99)*, June 30-July 2, Valencia, Spain.
Matsatsinis, N.F., P. Moraitis, V. Psomatakis and N. Spanoudakis (2000). Multi-agent architecture for agricultural products development, in: *Proceedings of the 2nd European Conference of the European Federation for Information Technology in Agriculture, Food and the Environment (EFITA 2000)*, September 27-30, Bonn, Germany.
Matsatsinis, N.F., P. Moraitis, V. Psomatakis and N. Spanoudakis (2001). An agent-based system for products penetration strategy selction, *Applied Artificial Intelligence Journal* (to appear).
Mihelis, G., E. Grigoroudis, Y. Siskos, Y. Politis and Y. Malandrakis (2001). Customer satisfaction measurement in the private bank sector, *European Journal of Operational Research*, 130 (2), 347-360.
Mousseau, V. and R. Slowinski (1998). Inferring an ELECTRE-TRI model from assignment examples, *Journal of Global Optimization*, 12 (2), 157-174.
Mousseau, V., J. Figueira and J.-Ph. Naux (2000a). Using assignment examples to infer weights for ELECTRE TRI method: Some experimental results, *European Journal of Operational Research*, 130 (2), 263-275.
Mousseau, V., R. Slowinski and P. Zielniewicz (2000b). A user-oriented implementation of the ELECTRE-TRI method integrating preference elicitation support, *Computers and Operational Research*, 27 (7-8), 757-777.
Murofushi, T. and M. Sugeno (1989). An interpretation of fuzzy measure and the Choquet integral as an integral withrespect to a fuzzy measure, *Fuzzy Sets and Systems* 29 (2), 201-227.
Oral, M. and O. Kettani (1989). Modelling the process of multiattribute choice, *Journal of the Operational Research Society*, 40 (3), 281–291.
Oral, M., O. Kettani, J.C. Cosset and M. Daouas (1992). An estimation model for country risk rating, *International Journal of Forecasting*, 8 (4), 583-593.
Pawlak, Z. (1982). Rough sets, *International Journal of Information and Computer Sciences*, 11 (5), 341-356.
Quinlan, J.R. (1986). Induction of decision trees, *Machine Learning*, 1 (1), 81-106.
Richard, J.L. (1983). Aide à la décision stratégique en P.M.E.", in: E. Jacquet-Lagrèze and Y. Siskos (eds.), *Méthode de décision multicritère*, Hommes et Techniques, Paris, 119-142.
Roy, B. (1985). *Méthodologie Multicritère d'Aide à la Décision*, Economica, Paris.

Sandalidou, E., E. Grigoroudis and Y. Siskos (2003). Organic and conventional olive oil consumers: A comparative analysis using customer satisfaction evaluation approach, in: G. Baourakis (ed.), Marketing trends for organic food in the advent of the 21st century, World Scientific (to appear).

Shakun, M.F. (1988). *Evolutionary systems design: Policymaking under complexity and group decision support systems*, Holden-Day, San Francisco, CA.

Shakun, M.F. (1991). Airline buyout: Evolutionary systems design and problem restructuring in group decision and negotiation, *Management Science*, 37 (10), 1291-1303.

Siskos, J. (1980). Comment modéliser les préférences au moyen de fonctions d'utilité additives, *RAIRO Recherche Opérationelle*, 14, 53-82.

Siskos, J. (1982). A way to deal with fuzzy preferences in multicriteria decision problems, *European Journal of Operational Research*, 10 (3), 314-324.

Siskos, J. (1983). Analyse de systèmes de décision multicritère en univers aléatoire, *Foundations of Control Engineering*, 10, (3-4), 193-212.

Siskos, J. (1985). Analyses de régression et programmation linéaire, *Révue de Statistique Appliquée*, 23 (2), 41-55.

Siskos, J. and D. K. Despotis (1989). A DSS oriented method for multiobjective linear programming problems, *Decision Support Systems*, 5 (1), 47-55.

Siskos, J. and N. Assimakopoulos (1989). Multicriteria highway planning: A case study, *Mathematical and Computer Modelling*, 12 (10-11), 1401-1410.

Siskos, J. and N.F. Matsatsinis (1993). A DSS for market analysis and new product design, *Journal of Decision Systems*, 2 (1), 35-60.

Siskos, J., A. Spyridakos and D. Yannacopoulos (1993). MINORA: A multicriteria decision aiding system for discrete alternatives, *Journal of Information Science and Technology*, 2 (2), 136-149.

Siskos, J., and C. Zopounidis (1987). The evaluation criteria of the venture capital investment activity: An interactive assessment, *European Journal of Operational Research*, 31 (3), 304-313.

Siskos, Y. (1986). Evaluating a system of furniture retail outlets using an interactive ordinal regression method, *European Journal of Operational Research*, 23, 179-193.

Siskos, Y. (2002). MUSTARD: Multicriteria utility-based stochastic aid for ranking decisions, *Journal of Behavioral Decision Making*, 15 (5), 461-465.

Siskos, Y. and A. Spyridakos (1999). Intelligent multicriteria decision support: Overview and perspectives, *European Journal of Operational Research*, 113 (2), 236-246.

Siskos, Y. and D. Yannacopoulos (1985). UTASTAR: An ordinal regression method for building additive value functions, *Investigação Operacional*, 5 (1), 39–53.

Siskos, Y., A. Spiridakos and D. Yannacopoulos (1999). Using artificial intelligence and visual techniques into preference disaggregation analysis: The MIIDAS system, *European Journal of Operational Research*, 113 (2), 236-246.

Siskos, Y., C. Zopounidis and A. Pouliezos (1994). An integrated DSS for financing firms by an industrial development bank in Greece, *Decision Support Systems*, 12 (2), 151-168.

Siskos, Y., E. Grigoroudis, C. Zopounidis and O. Saurais (1998). Measuring customer satisfaction using a collective preference disaggregation model, *Journal of Global Optimization*, 12 (2), 175-195.

Siskos, Y., E. Grigoroudis, N.F. Matsatsinis and G. Baourakis (1995a). Preference disaggregation analysis in agricultural product consumer behaviour, in: P.M. Pardalos, Y. Siskos and C. Zopounidis (eds.), *Advances in multicriteria analysis*, Kluwer Academic Publishers, Dordrecht, 185-202.

Siskos, Y., E. Grigoroudis, N.F. Matsatsinis, G. Baourakis and F. Neguez (1995b). Comparative behavioural analysis of European olive oil consumer, in: J. Janssen, C.H.

Skiadas and C. Zopounidis (eds.), *Advances in stochastic modelling and data analysis*, Kluwer Academic Publishers, Dordrecht, 293-310.

Siskos, Y., E. Grigoroudis, Y. Politis and Y. Malandrakis (2001). Customer satisfaction evaluation: Some real experiences, in: A. Colorni, M. Paruccini and B. Roy (eds.), *A-MCD-A: Multiple Criteria Decision Aiding*, European Commission Joint Research Centre, 297-314.

Siskos, Y., N.F. Matsatsinis and G. Baourakis (2001). Multicriteria analysis in agricultural marketing: the case of French olive oil market, *European Journal of Operational Research*, 130, (2), 315-331.

Siskos, Y., and E. Grigoroudis (2002). Measuring customer satisfaction for various services using multicriteria analysis, in: D. Bouyssou, E. Jacquet-Lagrèze, P. Perny, R. Słowiński, D. Vanderpooten and P. Vincke, P., (eds.), *Aiding decisions with multiple criteria: Essays in honor of Bernard Roy*, Kluwer Academic Publishers, Dordrecht, 457-482).

Slowinski, R. (1995). Rough set approach to decision analysis, *AI Expert Magazine*, 10 (3), 18-25.

Spiliopoulos, P. (1987). *Analyse et simulation du marché pour le lancement d'um nouveau produit – Réalisation d'un SIAD*, Thèse de 3e cycle, Universiré de Paris-Dauphine, Paris.

Spyridakos, A., Y. Siskos, D. Yannakopoulos and A. Skouris (2000). Multicriteria job evaluation for large organisations, *European Journal of Operational Research*, 130 (2), 375-387.

Srinivasan, V. and A.D. Shocker (1973). Linear programming techniques for multidimensional analysis of preferences, *Psychometrika*, 38 (3), 337–396.

Stam, A., M. Sun and M. Haines (1996). Artificial neural network representations for hierarchical preference structures, *Computers and Operations Research*, 23 (12), 1191-1201.

Stewart, T.J. (1987). Pruning of decision alternatives in multiple criteria decision making, based on the UTA method for estimating utilities, *European Journal of Operational Research*, 28 (1), 79-88.

Van de Panne, C. (1975). *Methods for linear and quadratic programming*, North-Holland Publishing Company, Amsterdam.

Wagner, H.M. (1959). Linear programming techniques for regression analysis, *Journal of the American Statistical Association*, 54, 206-212.

Young, F.W., J. De Leeuw, and Y. Takane (1976). Regression with qualitative and quantitative variables: An alternating least squares method with optimal scaling features, *Phychometrica*, 41 (4), 505-529.

Zaras, K. (2000). Rough approximation of a preference relation by a multi-attribute stochastic dominance for deterministic and stochastic evaluation problems, *European Journal of Operational Research*, 130 (2), 305-314.

Zionts, S. and J. Wallenius (1976). An interactive programming method for solving the multiple criteria problem, *Management Science*, 22 (6), 652-663.

Zopounidis C., N.F. Matsatsinis and M. Doumpos (1996). Developing a multicriteria knowledge-based decision support system for the assessment of corporate performance and viability: The FINEVA system, *Fuzzy Economic Review*, 1 (2), 35-53.

Zopounidis, C. (1987). A multicriteria decision-making methodology for the evaluation of the risk of failure and an application, *Foundations of Control Engineering*, 12 (1), 45-67.

Zopounidis, C. and M. Doumpos (1997). A multicriteria decision aid methodology for the assessment of country risk, *European Research on Management and Business Economics*, 3 (3), 13-33.

Zopounidis, C. and M. Doumpos (1998). Developing a multicriteria decision support system for financial classification problems: The FINCLAS system, *Optimization Methods and Software*, 8 (3-4), 277-304.

Zopounidis, C. and M. Doumpos (1999). Business failure prediction using UTADIS multicriteria analysis, *Journal of the Operational Research Society*, 50 (11), 1138-1148.

Zopounidis, C. and M. Doumpos (2000a). Building additive utilities for multi–group hierarchical discrimination: The MHDIS method, *Optimization Methods and Software*, 14 (3), 219–240.

Zopounidis, C. and M. Doumpos (2000b). INVESTOR: A decision support system based on multiple criteria for portfolio selection and composition, in: A. Colorni, M. Paruccini and B. Roy (eds.), *A-MCD-A: Multiple Criteria Decision Aiding*, European Commission Joint Research Centre, 371-381.

Zopounidis, C. and M. Doumpos (2000c) PREFDIS: A multicriteria decision support system for sorting decision problems, *Computers and Operations Research*, 27 (7-8), 779-797.

Zopounidis, C. and M. Doumpos (2001). A preference disaggregation decision support system for financial classification problems, *European Journal of Operation Research*, 130 (2), 402-413.

Zopounidis, C., Ch. Hurson and M. Doumpos (2000). *Risque-Pays: Evaluation des aspects economiques*, Sociaux et Politiques, Economica, Paris.

Zopounidis, C., M. Doumpos and S.H. Zanakis (1999). Stock evaluation using a preference disaggregation methodology, *Decision Sciences*, 30 (2), 313-336.