Decomposing the Persistence of Real Exchange Rates

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Abstract

We propose a new methodology for decomposing the persistence of deviations from Purchasing Power Parity (PPP). By directly comparing the impulse response function (IRF) of a VAR model, where the real exchange rate is Granger caused by a set of candidate variables, with the IRF of the equivalent ARMA model for the real exchange rate, we capture the effects of the Granger-causing variables on the half-life of deviations from PPP. Our empirical results for a set of 20 industrialised countries suggest that up to 62% of the persistence of real exchange rates can be attributed to interest rate differentials and relative business cycle position with the numenaire country.

Keywords: *real exchange rate; persistence measures; VAR; impulse response function; PPP.*

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1 Introduction

Long-run Purchasing Power Parity (PPP) states that real exchange rates, defined as the relative price of a basket of goods expressed in a common currency, should be stationary, implying that changes in the real exchange rate should be arbitraged away in the long run. Yet, one characteristic of real exchange rates is that they are highly persistent processes. In other words, the speed at which a given shock to the real exchange rate dissipates is very slow. One measure of persistence is half-life, defined as the number of periods required for a given shock to reduce to half its initial value. A large number of empirical studies has found that real exchange rates are stationary, but highly persistent processes with the consensus of half-lives of deviations from PPP between three and five years as Rogoff (1996) claimed.[1]

The majority of empirical studies compute half-lives of PPP deviations within a univariate framework, typically by estimating a first-order autoregressive model (AR(1)).[2] In such a specification, the error term, which accounts for the variation of the real exchange rate, can be thought of as a 'composite shock' that incorporates various individual shocks, such as monetary shocks or shocks to tastes and technology. As a result, impulse response analysis within the univariate framework cannot identify the effect of each individual shock, but simply tells us how fast the real exchange rate adjusts to a disturbance of unknown origins.

Various studies have attempted to measure the contributions of monetary and real shocks to the variability of the real exchange rate by estimating structural Vector Autoregressive (VAR) models. The results are mixed. Clarida and Gali (1994) find monetary shocks to be unimportant in contrast to aggregate demand shocks, which appear to be the major determinant of real exchange rate variability. On the other hand, Rogers (1999) reports that 20% to 60% of the variability of real exchange rates is attributable to monetary shocks.

[1] Taylor (2003, 2006) and Taylor and Taylor (2004) provide us with excellent literature reviews on the issue.

[2] Alternative methodologies include panel data models (see, inter alia, Frankel and Rose, 1996; Lothian, 1997; and Papell,1997) and nonlinear time series models (see, Taylor and Peel, 2000; Shintani, 2006; among others).

In this paper, we also employ a VAR methodology, although we take a different approach than decomposing the variance of the real exchange rate within a structural VAR framework. Rather than trying to identify the sources of structural shocks to the real exchange rate, we measure the relative contribution of a number of explanatory variables to the persistence of a shock on the real exchange rate by comparing the half-life estimates obtained from a VAR model with those obtained from the equivalent univariate ARMA models of the real exchange rate.[3] The VAR model includes apart from the real exchange rate, a set of explanatory variables that determine the short-run dynamics of the real exchange rate. By doing so, we are able to isolate the effect of these explanatory variables on the speed of adjustment of the real exchange rate towards the PPP level. By comparing the half-life of the VAR model with the half-life of the equivalent univariate model, we are able to measure directly the contribution of the explanatory variables to the persistence of the real exchange rate. For example, if the multivariate half-life estimates are not shorter than the univariate ones, then the explanatory variables cannot account, even partly, for the persistence of a shock on the real exchange rate.

In order to motivate our proposed methodology, let us first define the real exchange rate, y_{1t}, as the relative price of foreign goods in terms of domestic goods. In log form:

$$y_{1t} \equiv s_t - (p_t - p_t^*)$$

where s_t is the nominal exchange rate, measured in units of domestic currency per unit of foreign currency, and p_t (p_t^*) is the domestic (foreign) price index. Furthermore, let $Y_t = [y_{1t}, \mathbf{y}_{2t}]'$ be an $(n \times 1)-$vector of variables where $\mathbf{y}_{2t}$ is an $(n-1)$-vector of macroeconomic variables, which affect the dynamic adjustment of the real exchange rate towards the PPP level.

Let us further assume that Y_t follows a $n-$variate VAR(1) model.[4] It is well known that each variable in the VAR(1) model (including y_{1t}) has an equivalent univariate

[3] The methodology introduced in this paper can be used to measure the contribution of any (group of) explanatory variable(s) to the persistence of any variable of interest.

[4] The VAR(1) model is assumed at this stage for expositional purposes only. This assumption is relaxed in Section 2 that describes the methodology in a general framework.

ARMA$(n, n-1)$ representation, where n and $n-1$ are the maximum orders of the autoregressive and moving average parts, respectively (see Lutkepohl, 1993). In view of this 'equivalence', there is no specification error involved in one's decision to employ the ARMA model for estimating the response of y_{1t} to a unit shock in the error term, say e_t. The latter, however, is a combination of the errors in the VAR model, which in turn implies that the origins of this shock cannot be identified. Assume for simplicity that there is no contemporaneous correlation among the elements of Y_t, and consider the first equation of the VAR model, that is the one for the real exchange rate.[5] The error term in this equation, say ε_{1t}, describes the shocks in y_{1t} not accounted for by $\mathbf{y}_{2t}$, that is it describes the effects of any other random factors that affect y_{1t}. The VAR-response, IRF_m, of y_{1t} to a unit shock in ε_{1t} will, in general, be different from its equivalent ARMA-response, IRF_u, to a unit shock in e_t if the variables $\mathbf{y}_{2t}$ have actually a role to play. Indeed, the difference, $D = IRF_u - IRF_m$, describes the dynamic adjustment path of the real exchange rate which is solely due to the observed variables $\mathbf{y}_{2t}$. Obviously, the effects of other factors that influence y_{1t} not taken into account in the VAR specification are captured by IRF_u itself. The bigger D is, the more (less) important the role of $\mathbf{y}_{2t}$ (other factors) for the persistence of the y_{1t} will be.

To further clarify our point, assume that the half-life of y_{1t}, estimated within the ARMA model for y_{1t} is 20 quarters. On the other hand, assume that the half-life estimate obtained from the VAR model, which includes y_{1t} and $\mathbf{y}_{2t}$ is only 12 quarters. This means that the contribution of $\mathbf{y}_{2t}$ to the half-life of y_{1t} is 20-12=8 quarters. The remaining 12 quarters is the number of periods required for y_{1t} to adjust (by half) to shocks in other factors. In such a scenario, $\mathbf{y}_{2t}$ accounts for 40% (=8/20) of the persistence of y_{1t}.

Our methodology has three main advantages over the structural VAR approaches previously employed in the literature. First, it does not require to impose identifying restrictions in order to decompose innovations in the real exchange rate into structural shocks. Second, it allows to recover directly the effect of any variable on the persistence of PPP deviations from the impulse-response function, as opposed to the structural VAR approach,

[5] Once again, we consider the first equation of the VAR model for expositional purposes only. This assumption is relaxed in Section 2.

which focuses on the effect of some type of structural shocks on the variance of the real exchange rate. Third, it provides simple testable conditions on the estimates of *the* VAR which allow the researcher to assess the role of any candidate variable in determining the persistence of PPP deviations.

Our aim is to shed some light on the contribution of a set of macroeconomic variables, which are considered to be fundamental determinants of real exchange rates, to the persistence of deviations from PPP. This set of variables includes output growth differentials, short- and long-term interest rate differentials (both nominal and real) between the domestic and the foreign economy.

The remainder of the paper is structured as follows. Section 2 describes the methodology introduced in this paper and presents our main theoretical results. In Section 3 we provide an empirical illustration of our methodology by quantifying the relative importance of a set of macroeconomic variables on the persistence of the real exchange rate. Finally, Section 4 concludes this paper.

2 Impulse Response Analysis: Multivariate Models and their Equivalent Univariate Representations

This section highlights our methodology that aims at measuring the contribution of a group of explanatory variables to the persistence of a variable of interest. The methodology is based on the fact that the impulse response analysis within a VAR model differs in general from that conducted within the equivalent univariate ARMA models. The difference between the two IRFs reveals the contribution of the explanatory variables to the persistence of a shock on the variable of interest.

The analysis is based on a zero-mean, $n-$variate VAR(p) model.[6] Throughout this section, we assume that we want to measure the contribution of $z_t := (y_{1t}, .., y_{k-1t}, y_{k+1t}, .., y_{nt})'$ to the persistence of y_{kt}.[7] Thoughout this section, the subscript k is assumed to be fixed.

[6] The existence of a non-zero mean does not affect the impulse response analysis.

[7] In the empirical part of the paper (Section 3) y_{kt} stands for the real exchange rate.

Let $Y_t = (y_{1t}, y_{2t}, .., y_{nt})'$ follow a stable VAR(p) process:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + .. + A_p Y_{t-p} + U_t \quad (1)$$

where $A_m := [a_{ij,m}]$, $i, j = 1, 2, .., n$ and $m = 1, 2, .., p$ are $(n \times n)$ matrices of parameters. The error vector $U_t = (u_{1t}, u_{2t}, .., u_{nt})'$ is a white noise process, that is, $E(U_t) = 0$, $E(U_t U_t') = \Sigma_u := [\sigma_{ij}]$, $i, j = 1, 2, .., n$ and $E(U_t U_s') = 0$ for $t \neq s$. The covariance matrix Σ_u is assumed to be non-singular.

Before we proceed any further, it is important to emphasize the role of σ_{ik}, $i \neq k$ on the interpretation of the errors in the VAR model. If $\sigma_{ik} \neq 0$ for some $i \neq k$, then the error, u_{kt}, in the $k - th$ equation of the VAR model, cannot be interpreted as the innovations driving y_{kt}. On the other hand, if $\sigma_{ik} = 0$ for every $i \neq k$, then u_{kt} regains its status as 'the innovations' of y_{kt} in the VAR model and can be thought of as summarizing the factors that contribute to the variability of y_{kt}, other than y_{kt-i} and z_{t-i}, $i = 1, 2, .., p$.

Following Lutkepohl (1993), each component series y_{it}, $i = 1, 2, .., n$ of Y_t has an equivalent univariate ARMA($\overline{p}, \overline{q}$) representation where $\overline{p} \leq np$ and $\overline{q} \leq (n-1)p$.[8] For example, the equivalent univariate model for y_{kt} is:

$$y_{kt} = a_{1,k} y_{kt-1} + ... + a_{\overline{p},k} y_{kt-\overline{p}} + e_{kt} + \gamma_{1,k} e_{kt-1} + .. + \gamma_{\overline{q},k} e_{kt-\overline{q}} \quad (2)$$

where $a_{i,k}$, $i = 1, 2, .., \overline{p}$ and $\gamma_{i,k}$, $i = 1, 2, .., \overline{q}$ are functions of the VAR parameters $a_{ij,m}$ and σ_{ij} ($i, j = 1, 2, .., n$ and $m = 1, 2, .., p$). In view of this 'equivalence', there is no specification error involved in one's decision to employ the ARMA model for estimating the response of y_{kt} to a unit shock in the error term e_{kt}.

It is interesting to note that the MA error term, $w_{kt} \equiv e_{kt} + \gamma_{1,k} e_{kt-1} + .. + \gamma_{\overline{q},k} e_{kt-\overline{q}}$, is related to the original VAR error. The error in the univariate representation of y_{kt} can be thought of as an aggregation of the original errors in the VAR model. As a result, the variation of w_{kt} is due to the variation of any conbination of $u_{it}, i = 1, 2, ..., n$. Furthermore, the variance, σ_k^2, of the error term, e_{kt}, is a complicated function of the VAR parameters.

[8]For a proof, see Corollary 6.1.1. in Lutkepohl (1993), page 232.

This means that the shock e_{kt} of y_{kt} in the context of the ARMA model is determined by the structure of the intertemporal interactions between y_{kt} and z_t and the second moments of u_{kt} and $(u_{1t}, .., u_{k-1t}, u_{k+1t}, .., u_{nt})$. As a consequence, its 'origins' are far from clear.

Assume for simplicity that there is no contemporaneous correlation among the elements of Y_t, and consider the $k-th$ equation of the VAR model. The error term in this equation u_{kt} describes the shocks in y_{kt} not accounted for by z_t, that is it describes the effects of any other random factors that affect y_{kt}. The VAR-response, IRF_m, of y_{kt} to a unit shock in u_{kt} describes the effects of factors other than z_t that affect y_{kt}.[9] On the other hand, the IRF of the equivalent ARMA model, IRF_u, shows the effect of all the factors that influence y_{kt} (other than its own past values) on y_{kt}. Specifically, the difference, $D = IRF_u - IRF_m$, describes the persistence of y_{kt} which is solely due to the explanatory variables z_t. The size of D determines the importance of z_t for the persistence of y_{kt}.

Let us now compare the two IRFs, i.e.IRF_m and IRF_u, for two different cases. Section 2.1 examines the case where $\sigma_{ik} = 0$ for every $i \neq k$ and Section 2.2 examines the case where $\sigma_{ik} \neq 0$ for some $i \neq k$.[10]

2.1 The Case of a Diagonal Covariance Matrix, $\sigma_{ik} = 0$ for $i \neq k$

Throughout this subsection we assume $\sigma_{ik} = 0$ for $i \neq k$. The impulse response functions, IRF_m, in the context of the VAR(p) model is usually defined in the context of the infinite moving average representation of Y_t, that is $Y_t = \sum_{i=0}^{\infty} \Phi_i U_{t-i}$ where $\Phi_0 = I_n$ and $\Phi_i = \sum_{j=1}^{i} \Phi_{i-j} A_j$ for $i = 1, 2, ...$ ($A_j = 0$ for $j > p$). We want to examine the response of y_{kt} to a unit shock in its innovations. Note that $y_{kt} = FY_t$ where $F = \begin{bmatrix} 0 & .. & 0 & 1 & 0 & .. & 0 \end{bmatrix}$ is a $(1 \times n)$ matrix with a unit element in the $k-th$ column. Then, it is easy to show that

$$IRF_m(t) = F\Phi_t F' = F(\sum_{j=1}^{t} \Phi_{t-j} A_j)F' := \phi_{kk,t}$$

[9] In the case of the VAR model, a response in y_{kt} may be caused by an impulse in u_{it}, $i \neq k$ even if $\sigma_{ik} = 0$ $(i \neq k)$.

[10] The diagonality restrictions on the covariance matrix are tested in the empirical part of the paper for all the countries under consideration.

where $\phi_{kk,t}$ is the $k-th$ diagonal element of Φ_t.

Similarly, we can derive the infinite moving average representation of the equivalent univariate ARMA$(\overline{p},\overline{q})$ model for y_{kt}, say $y_{kt} = \sum_{i=0}^{\infty}\psi_{i,k}e_{kt-i}$, and calculate the impulse response function as follows:

$$IRF_u(t) = \psi_{t,k} \tag{3}$$

We are interested in comparing $IRF_u(t)$ with $IRF_m(t)$ in order to measure the contribution of z_t to the persistence of y_{kt}. It is straightforward to see that in general the two impulse response functions are different for finite t. Note that given the stability of (1), both IRF_u and IRF_m tend to zero as $t \longrightarrow \infty$.

However, there is one case where $IRF_u(t) = IRF_m(t)$ for every t. Specifically, this case arises when z_t does not Granger-cause y_{kt}. Hence:

Proposition 1 *In the context of (1) with $\sigma_{ik} = 0$ for $i \neq k$, $i = 1, 2, .., n$ and its equivalent univariate representation (2), when $z_t := (y_{1t}, .., y_{k-1t}, y_{k+1t}, .., y_{nt})'$ does not Granger-cause y_{kt}, i.e. when $a_{ki,m} = 0$ for $i \neq k$ and $m = 1, 2, .., p$, $IRF_u(t) = IRF_m(t)$ for every $t \geq 0$.*

Proof: See Appendix.

2.2 The Case of a Non-Diagonal Covariance Matrix, $\sigma_{ik} \neq 0$ for some $i \neq k$

Similarly to the previous case, the impulse response function for y_{kt} based on the equivalent univariate representation can be calculated by (3). On the other hand, the error term, u_{kt}, in the $k-th$ equation of the VAR(p) model does not coincide with the innovations driving y_{kt}. Following standard practice, we restore the orthogonality of the errors by utilizing the Cholesky decomposition of Σ_u, that is $\Sigma_u = PP'$, where P is a lower triangular matrix. We also define the following three matrices, (i) a diagonal matrix D with the same diagonal elements with P, (ii) $W = PD^{-1}$ and (iii) $\Lambda = DD'$. We can now obtain a different form of the Cholesky decomposition, that is $\Sigma_u = W\Lambda W'$. After some algebra, we can rewrite the VAR(p) model as follows:

$$Y_t = B_0 Y_t + B_1 Y_{t-1} + .. + B_{0p} Y_{t-p} + V_t \tag{4}$$

where $B_0 = I_n - W^{-1}$, $\mathrm{B}_i = W^{-1} A_i$, $i = 1, 2, .., p$ and $V_t = W^{-1} U_t$. This particular representation was obtained by assuming that y_{1t} is causally prior to all other components of Y_t. This means that the current values of y_{1t} do not react contemporaneously to changes in any of $y_{2t}, ..., y_{nt}$.[11] Similarly, this decomposition assumes that the $k-th$ equation can contain $y_{1t}, ..., y_{k-1t}$ but not $y_{kt}, ..., y_{nt}$. The error term, v_{kt}, in the $k-th$ equation of (4) is orthogonal to Y_{t-i}, $i = 1, 2, .., p$ and $y_{1t}, .., y_{k-1t}$, that is, it can be thought of as summarizing all the other factors that contribute to the variability of y_{kt}, apart from Y_{t-i}, $i = 1, 2, .., p$ and $y_{1t}, .., y_{k-1t}$. Once again, we define the impulse response function based on the following infinite MA representation of Y_t:

$$Y_t = \sum_{i=0}^{\infty} \Theta_i W_{t-i} \tag{5}$$

where $\Theta_i = \Phi_i P$ and the components of $W_t = (w_{1t}, w_{2t}, .., w_{nt})' := P^{-1} U_t$ are uncorrelated with variance-covariance matrix $\Sigma_W = I_n$.

We now define the Impulse Response Function, $IRF_{mo,}$ of y_{kt} to be the response of y_{kt} to a unit shock in its innovations, v_{kt}, after t periods. Therefore, $IRF_{mo}(k)$ is directly comparable to $IRF_u(k)$... We first need to note that $V_t = DW_t$. By construction, the diagonal element d_{ii} $(i = 1, 2, .., n)$ of D equals the diagonal element p_{ii} $(i = 1, 2, .., n)$ of P. Given that the element $\theta_{kk,t}$ of Θ_t in (5) measures the effect of a unit shock in w_{kt} on y_{kt} after t periods, we define IRF_{mo} to be:

$$IRF_{mo}(t) = \frac{\theta_{kk,t}}{p_{kk}}$$

The difference between $IRF_{mo}(t)$ and $IRF_u(t)$ measures the contribution of z_t and $y_{1t}, .., y_{k-1t}$ to the persistence of y_{kt}. It is straightforward to see that in general the two impulse re-

[11] Similarly to the methodology of variance decomposition, this is the typical problem of the ordering of the variables.

sponse functions are different for finite t.

Once again, there is one case where $IRF_u(t) = IRF_{mo}(t)$ for every t. Specifically, this case arises when z_t does not Granger cause y_{kt}. Hence, the following proposition holds:

Proposition 2 *In the context of (1) with $\sigma_{ik} \neq 0$ for some $i \neq k$, $i = 1, 2, .., n$ and its equivalent univariate representation (2), when z_t does not Granger cause y_{kt}, i.e. $a_{ki,m} = 0$ for $i \neq k$ and $m = 1, 2, .., p$, $IRF_u(t) = IRF_{mo}(t)$ for every $t \geq 0$.*

Proof: See Appendix.

3 An Application to Real Exchange Rate

We now implement the methodology described in the previous section to calculate the relative importance of a set of macroeconomic variables on the persistence of the real exchange rate. We first provide a rationale for the choice of the explanatory variables and then we present our findings.

3.1 Choice of Economic Variables

Economic theory has identified two main sets of determinants of real exchange rates: (a) real variables which describe the evolution of tastes and technology and determine the long-run equilibrium real exchange rate,[12] and (b) monetary/aggregate demand variables which describe the deviations of real exchange rates from PPP.[13]

While real disturbances, such as changes in tastes and technology, are likely to explain long-term changes in the real exchange rate, medium- and short-term changes are more likely to reflect monetary or aggregate demand shocks. Such shocks can have substantial effects on the real economy in the presence of short-term nominal price rigidities. This is a central feature of the Dornbusch (1976) sticky-price monetary model. In this model, monetary disturbances lead to overshooting of the real exchange rate due to short-term

[12] See, e.g. Balassa (1964) and Samuelson (1964). According to the so-called "Balassa-Samuelson hypothesis", the long-run equilibrium real exchange rate is determined by the share of nontradable goods in the consumer basket (i.e. by consumer preferences) and relative total factor productivity in the tradables and non-tradables sector.

[13] See, e.g. Dornbusch (1976, 1989) and Meese and Rogoff (1988).

price stickiness. During the adjustment to long-term equilibrium, deviations from PPP are related to output and interest rate differentials between the domestic and the foreign economy. Frankel (1979) derives an alternative representation of the real exchange rate in terms of real interest rate differentials.[14]

Guided by these theories, we choose GDP growth differentials, short- and long-term interest rate differentials (both nominal and real) between the domestic and the foreign economy as the main driving forces of real exchange rates.

A disclaimer is in order. It is clear that these variables capture a combination of both real and monetary disturbances making it difficult to relate them to any particular theory of exchange rate determination. For example, GDP growth differentials are related to the relative business cycle position between the domestic and the foreign economy but also reflect productivity differentials. Consequently, they capture a mixture of both monetary/aggregate demand disturbances and real disturbances.

Since it is very difficult in practice to proxy monetary and real disturbances with two orthogonal sets of variables, our empirical work does not aim at identifying the contribution of monetary and real shocks to the persistence of real exchange rates and, hence, at resolving the so-called "PPP puzzle". However, conditional on choosing the set of macroeconomic determinants of real exchange rates carefully, our methodology opens the way to directly test different theories of exchange rate determination.

3.2 Data

Our empirical analysis is based on post-1973, quarterly, real exchange rates for twenty countries.[15] Data for nominal exchange rates, consumer prices, short- and long-term interest rates and real GDP are collected from International Financial Statistics (IFS CD-Rom, December 2006).[16]

[14] In an empirical paper, Baxter (1994) finds a strong correlation between real exchange rates and real interest rate differentials.

[15] The countries employed are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Switzerland, Sweden and the UK. The sample period spans from 1973:Q1 to 2006:Q4.

[16] Nominal exchange rate: line ae.zf, long-term interest rate: line 61...zf, short term interest rate: line 60C...zf or 60B...zf, CPI: line 64...zf, real GDP: line 99BVRZF.

We consider twenty country pairs, with the US serving as the foreign country. The bilateral real exchange rate is measured as the nominal exchange rate, defined in units of domestic currency per dollar, multiplied by the ratio between the US and the domestic consumer price index. The business cycle position relative to the US is proxied by the 4-quarter real GDP growth differential between the home country and the US. Long-term interest rates are yields to maturity of 10-15 year government bonds and short-term interest rates are 3-month T-bills or money market rates depending on data availability. In order to compute real long-term interest rates, we subtract consumer price inflation over the past four quarters from the nominal yield. Although this method for computing real interest rates is not entirely satisfactory, since inflation is not measured over the term of the bond, it avoids problems of overlapping observations, compared with the method of computing true ex post real interest rates. Similarly, nominal short-term interest rates are made real by subtracting the inflation rate over the respective quarter. Interest rate differentials are calculated against the relative figures of the US which serves as the numenaire country in this analysis.

Inferences on the presence of unit roots in real exchange rates depends heavily on both the testing strategy and the sample employed. For example, Huizinga (1987) and Meese and Rogoff (1988) fail to reject the unit root null by means of standard unit root tests for the post-1973 period. The notorious low power of these tests may of course be the sole reason for not rejecting the null.[17] On the other hand, when longer-run time series are employed, blending fixed and floating rate data, the unit root hypothesis is rejected.[18] Similar evidence is obtained when the post-1973 data are expanded cross-sectionally, by means of panel data methods.[19] In the present case, the results from a variety of unit-root tests are, as usual, mixed.[20] *When the null hypothesis of stationarity is tested, the KPSS*

[17] Some recent results by Taylor (2001) forcefully point towards the 'low-power' interpretation of not rejecting the unit root null. Specifically, sampling the data at low frequencies makes it impossible to identify an adjustment process occurring at high frequencies, thus producing the false impression of long or even infinite half-lives. In another recent paper, Imbs *et al.* (2005) show that estimates of persistence of real exchange rates suffer from a positive cross-sectional aggregation bias.

[18] See, for example, Abuaf and Jorion (1990), Frankel (1990), Lothian and Taylor (1996) and Cheung and Lai (1998, 2000).

[19] See, for example, Wei and Parsley (1995), Frankel and Rose (1996), Higgins and Zakrajsek (1999).

[20] The unit root null is tested by means of the following tests: the standard Dickey-Fuller test (Dickey and Fuller, 1979), the Dickey-Fuller test with GLS detrending (Elliott *et al.*, 1996), the Point Optimal test

test fails to reject the null for the real exchange rates as well as the other macroeconomic variables for all the countries under consideration. When the null hypothesis of a unit root is tested, the standard Dickey-Fuller (DF) or Phillips-Perron (PP) tests typically fail to reject the null. The GLS versions of the DF tests, however, being more powerful than the standard DF tests, reject the unit root null in many cases.

The general picture emerging from the empirical literature and our own tests suggests treating real exchange rates and the candidate variables as having a highly persistent but ultimately stationary univariate representation.

3.3 Empirical Results

We begin by estimating a general VAR model that contains apart from the real exchange rate, two explanatory variables, that is the GDP growth differential and one of the available interest rate differentials (short- and long-term, real and nominal).[21] If typical Wald tests indicate that both explanatory variables are statistically significant for describing the behaviour of real exchange rate, we base the analysis on the estimated trivariate model. On the other hand, if our trivariate models suggest that either the growth differential or the interst rate differential measure does not influence the real exchange rate, we focus on finding a bivariate model that provides an adequate description of the dynamics of the real exchange rate.[22] In the event that no bivariate model improves on a univariate specification, we conclude that none of the available explanatory variables Granger causes the respective real exchange rate. Interestingly, our analysis points to seven countries that do not admit a multivariate specification. This is the case for Australia, Denmark, Finland, Greece, Italy, Spain and Sweden.[23] Taking this into account, we continue the analysis for the remaining thirteen countries.

(Elliott *et al.*, 1996), the Phillips-Perron test (Phillips and Perron, 1988) and the Ng-Perron test (Ng and Perron, 2001). The stationary null hypothesis is tested by means of the KPSS test (Kwiatkowski *et al.*, 1992). Results are available upon request.

[21] Some explanatory variables are not available for particular countries due to data limitations.

[22] We employ the Schwartz Information Criterion (SIC) to select the lag order of both the bivariate and trivariate VAR models. In all cases, SIC selects one lag. The only exception is Belgium where two lags are needed.

[23] Please note that the set of variables employed is not exhaustive. Other variables may prove significant for the decomposition of persistence of the real exchange rates in these countries.

Before proceeding to the calculation of the half-life of deviations from PPP in the VAR model, we test whether the contemporaneous correlation between innovations in the real exchange rate and other variables in each multivariate model is statistically different from zero. The importance of this condition was already discussed in Section 2. In the case of a zero correlation, we can compute the half-life using the original VAR innovations, otherwise our calculations should be based on the orthogonal transformation of the VAR innovations. In order to test this assumption, we estimated both a restricted and an unrestricted model and computed the Likelihood Ratio (LR) statistic. The results, reported in Table 1, suggest that the orthogonality restriction, i.e. zero contemporaneous correlation between the innovations in the real exchange rate and the other variables included in the VAR model holds for only six countries at a 10% significance level.

[INSERT TABLE 1]

We now turn on revealing the dynamic characteristics of the real exchange rates under consideration by examining the impulse response functions. In the cases that the orthogonality restriction between innovations in the real exchange rate and other variables is satisfied, we employ responses to a unit shock. In all other cases, we employ orthogonal impulse responses. It is important to note that when orthogonal IRFs are considered, these are dependent on the ordering of the variables. To ensure comparability of multivariate IRFs with the equivalent univariate IRFs, the real exchange rate is the first variable in the VAR model. Our results are reported in Table 2.

[INSERT TABLE 2]

The last column of Table 2 reports the explanatory variables employed in each country. Based on the model selection strategy outlined previously, trivariate VAR models were employed for Austria, Canada, Japan, New Zealand, Norway, Portugal, Switzerland and the UK, while bivariate ones for Belgium, France, Germany, Ireland and the Netherlands. As already mentioned, seven of the countries employed do not admit a VAR representation along the lines of our study. Estimated half-lives from the VAR models along with their Monte Carlo 95% confidence intervals are presented in Columns 6 and 7, respectively. The respective figures range from 5 quarters (Ireland) to 21 quarters (Canada). Interestingly, the confidence intervals have finite bounds in all cases with Germany and the UK generating the tightest intervals. In these cases, even the upper bound hovers at around 2 years. Consistent with other studies (see, e.g. Shintani, 2006), the greater upper bound, though finite, is detected for Canada (53 quarters).

Next, we turn to our main focus, which is the contribution of the explanatory variables included in the VAR model to the persistence of the real exchange rates. In order to do this, we need to estimate the equivalent univariate model for each one of the real exchange rate series. As noted in Section 2, the order of the equivalent ARMA model depends on the lag order of the VAR model and the number of variables included in the VAR. Thus, in cases of trivariate VAR(1) models, the equivalent univariate model is ARMA(3,2), while in cases for bivariate VAR(1) models, the equivalent univariate model is ARMA(2,1). Finally, in the case of Belgium where the estimated VAR model is bivariate with two lags, the corresponding univariate model is ARMA(4,2). The lag order of the equivalent ARMA models, along with the estimated half-lives and their 95% simulated confidence intervals are reported in columns 3, 4 and 5 respectively (Table2). For comparison purposes, we also report the estimated half-life of the univariate AR(1) model in the second column of Table 2. Our results show that the half-lives in the context of the ARMA models are higher than those of the VAR models, revealing that the explanatory variables contribute to the persistence of real exchange rate. Specifically, ARMA estimates of half-lifes range from 7 quarters (Switzerland) to 26 quarters (Portugal). In this case, the median half-life is 12 quarters compared to the respective figure of 8.5 quarters in the case of VAR models. It is worth mentioning that our VAR methodology produces tighter confidence intervals than the respective univariate ones in the majority of the cases.

We can now measure the contribution of the explanatory variables included in the VAR models to the persistence of real exchange rate by simply comparing the estimated half-lives of the VAR models and the equivalent ARMA models. We compute the fraction of half-life attributable to the set of macroeconomic variables included in the VAR model as (HL_u -HL_v)/HL_u, with HL_u and HL_v denoting the half-lifes of the univariate and VAR models, respectively. Column 7 of Table 2 tabulates this measure of contribution to the peristence of real exchange rates. On average, the explanatory variables included in this analysis contribute around 30% to the persistence of real exchange rate. These contributions range from just 6% in the case of Belgium to an impressive 62% in the case of Portugal.

4 Conclusions

In this paper, we estimated the half-life of PPP deviations in the context of a Vector Autoregressive model, where the real exchange rate is allowed to interact with a set of macroeconomic variables, suggested by theories of exchange rate determination. By doing this, we were able to discern the relative effect of these variables on the speed of adjustment of the real exchange rate towards long-run PPP. We first showed that the impulse response function of a variable participating in the VAR model is not, in general, the same with the impulse response function obtained from the equivalent ARMA representation of this variable, if the latter is Granger caused by other variables in the system. The difference between the two impulse response functions captures the effect of the Granger-causing variables on the dynamic adjustment process of the variable of interest.

We investigated the implications of our analytical results for the speed of adjustment of twenty real exchange rates vis-a-vis the US dollar during the post-Bretton Woods period. Our empirical results suggest that real exchange rates are in fact Granger caused by these variables in the majority of cases. As a result, the adjustment horizons of deviations from PPP decrease substantially. The median half-life estimate across the pairs of real exchange rates is around two years, suggesting that real or nominal interest rate differentials and GDP growth differentials account for a significant fraction of deviations from PPP. Comparing the half-life estimates of the equivalent univariate models with the half-life estimates of the VAR model, we conclude that on average 30% of the half-life of deviations from PPP is due to these variables.

Of course, although real or nominal interest rate differentials and GDP growth differentials explain a significant fraction of deviations from PPP, our results leave a good bit of variation in real exchange rates to unknown sources. These sources still account on average for a half-life of two years, hence, a puzzle remains as to whether real sources are volatile enough to explain the observed movements of real exchange rates. However, recent work on the PPP puzzle suggests that standard methods of estimation used in the literature largely overestimate the size of real exchange rates half-lives because they fail to correct for a number of biases stemming from parameter heterogeneity, temporal aggregation and nonlinear adjustment.

Our method is not able to identify whether the persistence of real exchange rates is due to real or monetary shocks and, hence, does not address the so-called "PPP puzzle". However, it opens the way to assess the role of fundamental determinants of real exchange rates identified by different theories on the persistence of deviations from PPP. Further work is needed to address the issue of identification. Finally, our method is general enough to assess the importance of fundamental determinants on the observed persistence of a wide range of economic and financial variables, such as inflation, real wages, dividend-price ratios etc.

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Appendix

Remark 1: Let $A := [a_{ij}]$ and $B := [b_{ij}]$ be two $(n \times n)$ matrices where $a_{ki} = b_{ki} = 0$ for some k and $i \neq k$. Define $\Gamma := [\gamma_{ij}] = A + B$ and $\Delta := [\delta_{ij}] = AB$. Then, $\gamma_{ki} = \delta_{ki} = 0$ for $i \neq k$.

Proof. For $i \neq k$, $\gamma_{ki} = a_{ki} + b_{ki} = 0 + 0 = 0$.

Similarly for $i \neq k$, $\delta_{ki} = \sum_{m=1}^{n} a_{km} b_{mi} = 0$ since $a_{km} = 0$ when $m \neq k$ and when $m = k$, $b_{mi} = b_{ki} = 0$ since $i \neq k$. ■

Proposition 1

Proof. First of all, it is straightforward to see that when z_t does not Granger cause y_{kt}, the equivalent univariate model for y_{kt} coincides with the $k-th$ equation of (1):

$$y_{kt} = a_{kk,1} y_{kt-1} + ... + a_{kk,p} y_{kt-p} + u_{kt}$$

and the impulse response function is:

$$IRF_u(t) = \psi_{t,k} = \sum_{j=1}^{t} \psi_{t-j,k} a_{kk,j}$$

where $\psi_{0,k} = 1$ and $a_{kk,j} = 0$ for $j > p$.

Let us now calculate the impulse response function in the context of the multivariate VAR(p) model. Let $\phi_{ij,m}$ be the element of Φ_m in the $i-th$ row and $j-th$ column. Then, by definition of the Φ_m $(m = 0, 1, ...)$ matrix:

$$\phi_{ki,t} = 0 \text{ for } i \neq k \text{ and } t \in N \tag{6}$$

To see this, remember that $\Phi_0 = I_n$ and thus $\phi_{ki,0} = 0$ for $i \neq k$. Furthermore, $\Phi_1 = A_1$ and thus $\phi_{ki,1} = 0$ for $i \neq k$ (since $a_{ki,1} = 0$ for $i \neq k$). In general, $\Phi_t = \sum_{j=1}^{t} \Phi_{t-j} A_j$, $t = 1, 2, ...$ is the sum of t products of two matrices Φ_{t-j} and A_j where $\phi_{ki,t-j} = a_{ki,j} = 0$ for $i \neq k$. Then, according to Remark 1, $\phi_{ki,t} = 0$ for $i \neq k$.

Given that $a_{ki,t} = \phi_{ki,t} = 0$ for every $t \in N$ and $i \neq k$, it is easy to see that:

$$IRF_m(t) = \phi_{kk,t} = \sum_{j=1}^{t} \phi_{kk,t-j} a_{kk,j}$$

By comparing the two impulse response functions, we can see that $IRF_u(t) = IRF_m(t)$ for every $t \in N$ if-f $\psi_{t,k} = \phi_{kk,t}$ for every $t \in N$. We prove this by induction. For $t = 0$, $\psi_{0,k} = \phi_{kk,0} = 1$. For $t = 1$, $\psi_{1,k} = \phi_{kk,1} = a_{kk,1}$. Assume that $\psi_{t,k} = \phi_{kk,t}$ for every $t < m$. We need to show that $\psi_{m,k} = \phi_{kk,m}$. $\psi_{m,k} = \sum_{j=1}^{m} \psi_{m-j,k} a_{kk,j} = \sum_{j=1}^{m} \phi_{kk,m-j} a_{kk,j} = \phi_{kk,m}$.
■

Proposition 2

Proof. First of all, it is straightforward to see that when z_t does not Granger cause y_{kt}, the equivalent univariate model for y_{kt} coincides with the $k-th$ equation of (1):

$$y_{kt} = a_{kk,1} y_{kt-1} + ... + a_{kk,p} y_{kt-p} + u_{kt}$$

and the impulse response function is:

$$IRF_u(t) = \psi_{t,k} = \sum_{j=1}^{t} \psi_{t-j,k} a_{kk,j}$$

where $\psi_{0,k} = 1$ and $a_{kk,j} = 0$ for $j > p$.

Let us now calculate the impulse response function in the context of the orthogonal multivariate VAR(p) model. Given that z_t does not Granger cause y_{kt} and in a similar way with the proof of proposition 1, we can prove that $\phi_{ki,t} = 0$ for $i \neq k$.

$\Theta_t = \Phi_t P$. This means that $\theta_{kk,t} = \sum_{j=1}^{n} \phi_{kj,t} p_{jk} = \phi_{kk,t} p_{kk}$. Thus, $IRF_{mo}(t) = \frac{\theta_{kk,t}}{p_{kk}} = \frac{\phi_{kk,t} p_{kk}}{p_{kk}} = \phi_{kk,t}$. As a result, $IRF_{mo}(t) = IRF_m(t)$ and based on proposition 1, we get that $IRF_{mo}(t) = IRF_u(t)$ for every finite $t \geq 0$. ■

Table 1. Orthogonality Restrictions				
	Log Likelihood			
	Unrestricted	Restricted	LR-statistic	p-value
Australia	– – –	– – –	– – –	– – –
Austria	-108.577	-113.052	8.949	0.003
Belgium	92.758	88.283	8.950	0.003
Canada	115.740	115.501	0.477	0.490
Denmark	– – –	– – –	– – –	– – –
Finland	– – –	– – –	– – –	– – –
France	50.640	45.513	10.253	0.001
Germany	86.553	85.993	1.120	0.290
Greece	– – –	– – –	– – –	– – –
Ireland	-19.630	-20.067	0.874	0.350
Italy	– – –	– – –	– – –	– – –
Japan	-275.491	-276.354	1.726	0.189
Netherlands	95.018	93.573	2.890	0.089
N.Zealand	-161.733	-163.600	3.726	0.054
Norway	-187.347	-189.953	5.212	0.022
Portugal	-225.410	-225.473	0.127	0.722
Spain	– – –	– – –	– – –	– – –
Switzerland	-166.870	-169.616	5.493	0.019
Sweden	– – –	– – –	– – –	– – –
UK	-200.582	-201.841	2.523	0.112

Table 2: Estimated half-lives and 95% confidence intervals

	AR(1)	ARMA			VAR		%	Explanatory
	h	$(\overline{p}, \overline{q})$	h	95% c.i.	h	95% c.i.	contribution	variables
Australia	20	–	–	– – –	– – –	– – –	– – –	– – –
Austria	15	$(3, 2)$	15	$(2, 29)$	8	$(3, 13)$	47	nlid,gd
Belgium	17	$(4, 2)$	16	$(4, 33)$	15	$(4, 28)$	6	nlid
Canada	32	$(3, 2)$	24	$(5, 28)$	21	$(6, 53)$	13	nlid,gd
Denmark	14	–	–	–	–	–	–	–
Finland	11	–	–	–	–	–	–	–
France	14	$(2, 1)$	12	$(3, 22)$	9	$(3, 17)$	25	rlid
Germany	14	$(2, 1)$	12	$(3, 18)$	6	$(3, 8)$	50	nlid
Greece	16	–	–	–	–	–	–	–
Ireland	7	$(2, 1)$	8	$(2, 12)$	5	$(2, 8)$	38	rlid
Italy	14	–	–	–	–	–	–	–
Japan	16	$(3, 2)$	14	$(3, 22)$	11	$(4, 21)$	21	rsid,gd
Netherlands	12	$(2, 1)$	11	$(3, 17)$	6	$(3, 9)$	46	nlid
N.Zealand	9	$(3, 2)$	9	$(2, 11)$	8	$(3, 12)$	11	nsid,gd
Norway	12	$(3, 2)$	12	$(3, 20)$	9	$(4, 14)$	25	nlid,gd
Portugal	25	$(3, 2)$	26	$(2, 78)$	10	$(3, 24)$	62	rsid,gd
Spain	16	–	–	–	–	–	–	–
Switzerland	12	$(3, 2)$	7	$(2, 13)$	6	$(3, 10)$	14	nsid,gd
Sweden	25	–	–	–	–	–	–	–
UK	10	$(3, 2)$	10	$(2, 17)$	7	$(4, 9)$	30	nsid,gd

Notes: h: half-life, *c.i.* : confidence interval, *nlid*: nominal long-term interest rate differential, *nsid*: nominal short-term interest rate differential, *rlid*: real long-term interest rate differential, *rsid*: real short-term interest rate differential, *gd*: growth differential