Measuring Risk Aversion across Countries from the Consumption-CAPM: A Spectral Approach

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Abstract

By using the Consumption-CAPM, Campbell (2003) reports cross-country evidence that imply implausibly large coefficients of relative risk aversion, thus confirming the equity premium puzzle in an international context. In this paper we adopt a spectral approach to re-estimate the values of risk aversion over the frequency domain. Our findings indicate that at lower frequencies risk aversion falls substantially across countries, thus yielding in many cases reasonable values of the implied coefficient of risk aversion.

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1. Introduction

A persistent puzzle in the macroeconomics and finance literature has been the failure of the Consumption Capital Asset Pricing Model (C-CAPM), which measures risk by non-durables consumption beta, to provide plausible levels for the coefficients of relative risk aversion in the US.[1] In a detailed cross-country study, Cambpell (2003) reports evidence from 11 countries that imply extremely high values of risk aversion, which usually exceed many times the value of 10 considered plausible by Mehra and Prescott (1985), and claims "*...that the equity premium puzzle is a robust phenomenon in international data*".

Using the same theoretical setup and dataset as in Campbell (2003), we adopt a spectral approach to re-estimate the values of risk aversion over the frequency domain.[2] Our findings indicate that at lower frequencies risk aversion falls substantially across countries, thus yielding in many cases reasonable values of the implied coefficient of risk aversion.

2. Measuring risk aversion over the frequency domain

We follow Campbell (2003) and we assume a representative investor who faces an intertemporal choice problem in complete and frictionless capital markets. The representative investor maximizes a time-separable utility function, $U(C_t)$, in consumption, C. The solution to this problem yields the following Euler condition:

$$U'(C_t) = \delta E_t[(1 + R_{i,t+1})U'(C_{t+1})] \tag{1}$$

where δ is the discount factor and $(1 + R_{i,t+1})$ represents the gross rate of return on asset i. Employing a time-separable power utility function $U(C_t) = \sum_{j=0}^{\infty} \delta^j \frac{C_{t+j}^{1-\gamma}}{1-\gamma}$ where γ is the coefficient of relative risk aversion, we get from (1) that:

$$E_t[(1 + R_{i,t+1})\delta(\frac{C_{t+1}}{C_t})^{-\gamma} = 1 \tag{2}$$

Following Hansen and Singleton (1983), we assume that the joint conditional distribution of

[1] Mehra (2003) and Cochrane (2005) provide extensive surveys of the relevant literature.

[2] Some studies have investigated the implications of spectral analysis within economic applications, mostly by interpretating (high) low-frequency estimates as the (short) long-run component of the relationship under scrutiny; see, for instance, Engle (1974, 1978).

asset returns and consumption is lognormal. With time-varying volatility we get after taking logs that:

$$E_t r_{i,t+1} + \log\delta - \gamma E_t[\Delta c_{t+1}] + 1/2(\sigma_i^2 + \gamma^2\sigma_c^2 - 2\gamma\sigma_{i,c}) = 0 \tag{3}$$

where $c_t \equiv \log(C_t)$, $r_{i,t} \equiv \log(1 + R_{i,t})$, and σ_i^2 and σ_c^2 denote the unconditional variances of log stock return innovations and log consumption innovations respectively, and $\sigma_{i,c}$ represents the unconditional covariance of innovations between log stock returns and consumption growth. Letting then $e_{i,t+1} \equiv E_t[r_{i,t+1} - r_{f,t+1}]$ denote the excess return over the riskfree rate, $r_{f,t+1}$, we get that the excess return on any asset over the riskless rate is constant and therefore the risk premium on all assets is linear in expected consumption growth with the slope coefficient, γ, given by:

$$\gamma = \frac{e_{i,t+1} + 0.5\sigma_i^2}{\sigma_{i,c}} \tag{4}$$

Now, departing from the time domain to the frequency domain we can rewrite (4) for each frequency.[3] After dropping the time subscript for simplicity, we get that the coefficient of risk aversion over the whole band of frequencies ω, where ω is a real variable in the range $0 \preceq \omega \preceq \pi$, is given by:

$$\gamma_\omega = \frac{e + 0.5 f_{ee}(\omega)}{f_{ec}(\omega)} \tag{5}$$

where e denotes the excess log return of the stock market over the risk-free rate. As is well known, the cross-spectrum, $f_{ec}(\omega)$, between e and c is complex-valued and can be decomposed into its real and imaginary components, given here by:

$$f_{ec}(\omega) = C_{ec}(\omega) - iQ_{ec}(\omega), \tag{6}$$

[3] In general, the spectrum of a process, say x_t, can be written as $f_{xx}(\omega) = \rho_0 + 2\sum_{k=1}^{\infty}\rho_k\cos(k\omega)$, where ρ_k is the k-order autocovariance function of the series. In turn, we can consider the multivariate spectrum, $F_{yx}(\omega)$, for a bivariate zero mean covariance stationary process $Z_t = [y_t, x_t]^\top$ with covariance matrix $\Gamma(\cdot)$, which is the frequency domain analog of the autocovariance matrix. The diagonal elements of $F_{yx}(\omega)$ are the spectra of the individual processes, $f_{yy}(\omega)$ and $f_{xx}(\omega)$, while the off-diagonal ones refer to the cross-spectrum or cross spectral density matrix of y_t and x_t. In detail, $F_{yx}(\omega) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\Gamma(k)e^{-ik\omega} = \begin{bmatrix} f_{xx}(\omega) & f_{yx}(\omega) \\ f_{xy}(\omega) & f_{yy}(\omega) \end{bmatrix}$, where $F_{yx}(\omega)$ is an Hermitian, non-negative definite matrix, i.e. $F_{yx}(\omega) = F_{yx}^*(\omega)$, with $*$ denoting the complex cojugate transpose since $f_{yx}(\omega) = \overline{f_{xy}(\omega)}$. See Hamilton (1994) for a more detailed presentation of spectral analysis.

where $C_{ec}(\omega)$ denotes the *co-spectrum* and $Q_{ec}(\omega)$ the *quadrature spectrum.* The measure of co-movement between returns and consumption over the frequency domain is then given by:

$$c_{ec}^2(\omega) \equiv \frac{|f_{ec}(\omega)|^2}{f_{ex}(\omega) f_{cc}(\omega)} = \frac{C_{ee}^2 + Q_{cc}^2}{f_{ee}(\omega) f_{cc}(\omega)} \tag{7}$$

where $0 \leq c_{ec}^2(\omega) \leq 1$ is the squared *coherency*, which provides an measure of the correlation between the two series at each frequency and can be interpreted intuitively as the frequency-domain analog of the correlation coefficient.[4]

The spectra and co-spectra of a vector of time-series for a sample of T observations can be estimated for a set of frequencies $\omega_n = 2\pi n/T$, $n = 1, 2, ..., T/2$. The relevant quantities are estimated through the periodogram, which is based on a representation of the observed time-series as a superposition of sinusoidal waves of various frequencies; a frequency of π corresponds to a time period of two quarters, while a zero frequency corresponds to infinity.[5]

3. Empirical findings

To calculate the coefficient of risk aversion from (5) we use the Campbell (2003) dataset, which combines quarterly data for consumption, interest rates and prices. We present our estimates only for the countries for which at least 100 observations are available in the dataset, namely Australia (1970:1-1998:4), Canada (1970:1-1998:4), France (1973:2-1998:3), Italy (1971:2-1998:1), Japan (1970:2-1998:4), Sweden (1970:1-1999:2), UK (1970:1-1999:1), and the US (1947:2-1998:3 and 1970:1-1998:3). To allow for a direct comparison with the evidence in Campbell (2003), we present two measures of risk aversion. The first one, termed RRA(1), is calculated directly from (5), whereas the second one, denoted by RRA(2), assumes a unitary correlation of excess returns with consumption growth. Although this is a counterfactual exercise, we follow closely Campbell (2003) and we postulate a unitary elasticity between returns and consumption growth to account for the sensitivity of the implied risk aversion on the smoothness of consumption rather than its

[4]Engle (1976) gives an early treatment on the frequency-domain analysis and its time-domain counterpart.

[5]Consistent estimates of the spectral matrix can be obtained by either smoothing the periodogram, or by employing a lag window approach that both weighs and limits the autocovariances and cross-covariances used. We use here the Bartlett window, which assigns linearly decreasing weights to the autocovariances and cross-covariances in the neighborhood of the frequencies considered and zero weight thereafter, with the lag, k, set using the rule $k = 2\sqrt{T}$, as suggested by Chatfield (1989).

low correlation with excess returns. We then identify the short-run estimates of risk aversion as the averages of fluctuations corresponding from 2 to 6 quarters in the time domain, the medium-run (or business cycle) estimates as the averages of fluctuations from 8 to 32 quarters, whereas the long-run estimates are derived from the averages of oscillations with duration above 32 quarters.

Table 1 presents the short-run spectral estimates of the variabilities of excess log stock return, σ_e^2, consumption growth, σ_c^2, the squared coherency c_{ec}^2, and the implied coefficients of relative risk aversion, γ_ω. To facilitate the exposition we report next to the country name the values of relative risk aversion from Table 4 in Campbell (2003), termed $\mathrm{RRA}_c(1)$ and $\mathrm{RRA}_c(2)$ respectively. Our results from the spectral analysis at high frequencies corroborate the evidence by Campbell (2003) suggesting that risk aversion at high frequencies is found to be extremely large (with the possible exception of Italy). This picture continues to hold under the assumption of a unitary elasticity between excess returns and consumption growth and is in line with the findings typically reported in the literature on the C-CAPM.

Table 2 performs the same exercise for the medium-run frequencies. As the time horizon increases, risk aversion is in general lower, but is still found to be implausibly high exceeding the value of 10, even when a unitary correlation is imposed. Thus we find that the equity premium puzzle persists at business-cycle frequencies.

The performance of the C-CAPM improves substantially at low frequencies. As shown in Table 3, the coefficients of risk aversion now range between 5.0 (Australia) to 28.5 (Sweden). When a unitary correlation coefficient is imposed, these estimates are slightly reduced for all the countries at hand and range from 4.1 to 28.1. This improvement in the low-frequency estimates of relative risk aversion is driven by the spectral properties of the data at hand. As we move to lower frequencies, the variability of consumption growth, σ_c^2, increases significantly, reaching even 10 times its high-frequency value matching the variability of log excess returns, σ_e^2, and as such the covariance of returns and consumption increases. This property is coupled for most of the countries at hand with a rise in the estimated coherency betweeen consumption and returns.[6]

[6]It is worth mentioning that the coherency is not maximised at the lowest frequency for all countries. The coherency reaches its maximum in the short-run (2 to 6 quarters) in Italy and Japan and in the medium-run (2-4 years) in Australia and Canada.

4. Conclusions

The paper attempts to re-address the empirical issue of implausibly high risk aversion within the context of the C-CAPM by looking at the pattern of risk aversion over the frequency domain. Our results show that as lower frequencies are taken into account, risk aversion falls substantially across countries and, in many cases, is consistent with more reasonable values of the coefficient of risk aversion. This evidence shows some improvement towards understanding the dynamics of the C-CAPM by reconciling its standard single-factor version with lower values of risk aversion and thus the equity premium over the frequency domain appears to be less of a puzzle. However, we emphasize that the point estimates of long-run risk aversion still appear to be relatively high and a more in-depth analysis is warranted to align the model with reasonable coefficients of risk aversion.

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Table 1. Short-run cross-country estimates of risk aversion

Country	$\text{RRA}_c(1)$	$\text{RRA}_c(2)$	$\sigma_e^2 = f_{ee}$	$\sigma_c^2 = f_{cc}$	c_{ec}^2	RRA(1)	RRA(2)
Australia	58.4	8.4	0.00115	0.00001	0.63	33.3	26.5
Canada	59.3	12.0	0.00107	0.00001	0.65	89.0	71.6
France	<0	12.3	0.00210	0.00002	0.70	98.2	82.3
Italy	<0	10.4	0.00295	0.00001	0.65	21.0	16.9
Japan	82.6	9.3	0.00143	0.00002	0.73	53.6	45.8
Sweden	1713.2	26.5	0.00199	0.00001	0.62	195.5	154.4
UK	186.0	17.2	0.00149	0.00002	0.68	134.7	110.7
US1	240.6	49.3	0.00107	0.00000	0.43	449.9	296.5
US2	150.1	41.2	0.00119	0.00000	0.54	436.3	319.6

Notes:
1) RRAc(1) and RRAc(2) denote the estimates of risk aversion reported by Campbell (2003).
2) See the text for the definitions of σ_e^2, σ_c^2, σ_{ec}^2 .
3) US1 refers to the sample starting at 1947:2 and US2 at the sample starting at 1970:1.

Table 2. Medium-run cross-country estimates of risk aversion

Country	$\sigma_e^2 = f_{ee}$	$\sigma_c^2 = f_{cc}$	c_{ec}^2	RRA(1)	RRA(2)
Australia	0.00231	0.00000	0.74	44.3	38.1
Canada	0.00182	0.00001	0.76	45.4	39.6
France	0.00160	0.00003	0.65	97.3	78.5
Italy	0.00399	0.00001	0.62	21.9	17.3
Japan	0.00185	0.00001	0.67	67.6	55.3
Sweden	0.00221	0.00000	0.59	383.9	294.3
UK	0.00234	0.00001	0.63	149.5	118.5
US1	0.00154	0.00000	0.80	270.9	242.7
US2	0.00123	0.00000	0.68	245.3	202.2

Notes:
1) See Table 1.

Table 3. Long-run cross-country estimates of risk aversion

Country	$\sigma_e^2 = f_{ee}$	$\sigma_c^2 = f_{cc}$	c_{ec}^2	RRA(1)	RRA(2)
Australia	0.00295	0.00033	0.68	5.0	4.1
Canada	0.00209	0.00029	0.63	10.6	8.4
France	0.00423	0.00017	0.84	22.1	20.3
Italy	0.00426	0.00031	0.26	5.2	2.6
Japan	0.00125	0.00058	0.68	10.3	8.5
Sweden	0.00539	0.00014	0.97	28.5	28.1
UK	0.00406	0.00034	0.97	16.2	15.9
US1	0.00287	0.00036	0.87	20.4	19.1
US2	0.00430	0.00025	0.89	16.2	15.3

Notes:
1) See Table 1.