Consumption Risk over the Frequency Domain

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Abstract

In this paper we evaluate the Consumption Capital Asset Pricing Model by employing as an explanatory variable consumption risk over the frequency domain. We modify the standard two-step methodology (Fama and French, 1992) to account for the spectral properties of consumption risk and we find that lower frequencies of consumption risk explain up to 98% of the cross-sectional variation of expected returns and that the equity premium puzzle is resolved. These results are robust to the definitions of the variables, the sample span and the set of portfolios utilized, and the maturity of interest rates.

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1. Introduction

A persistent puzzle in the macroeconomics and finance literature has been the failure of the Consumption Capital Asset Pricing model (C-CAPM), which measures risk by consumption beta, first to explain empirically the differences in expected stock returns by the variation in the covariance of consumption and returns, and second to provide plausible levels of risk aversion. However, several studies have emphasized that this puzzle is likely to emerge primarily at short horizons due for instance to habits, information delays, and transaction costs, and that short-run consumption should be replaced with a portfolio that exhibits higher correlations with long-run movements in consumption.\(^1\) Singleton (1990) has first pointed out that the inclusion of consumption growth over longer time intervals improves the empirical fit of the single-good asset pricing model. Brainard et al. (1991) have shown that the performance of the C-CAPM improves as the horizon increases, a finding confirmed by Daniel and Marshall (1997) who have found that at lower frequencies aggregate returns and consumption growth are more correlated and the behavior of the equity premium becomes less puzzling. In a series of papers, Parker (2001, 2003) and, in particular, Parker and Julliard (2005) have allowed for long-term consumption dynamics by focusing on the ultimate risk to consumption, defined as the covariance between an asset’s return during a quarter and consumption growth over the quarter of the return and several following quarters, and have found that it explains the cross-sectional variation in returns surprisingly well, but also show that the equity premium puzzle persists and high levels of risk aversion are required to line up the model with the data. Bansal and Yaron (2004), Bansal et al. (2005), and Hansen et al. (2008) show that when consumption risk is measured by the covariance between long-run cashflows from holding a security and long-run consumption growth in the economy, the differences in consumption risk provide useful information about the expected return differentials across assets.

In this paper we assess the explanatory power of consumption risk over the frequency domain by performing within the C-CAPM a dynamic analysis of consumption risk over the spectrum rather than over the time domain. As pointed out by Granger and Hatanaka back in 1964, according to the spectral representation theorem a time series can be seen as the sum of waves of different periodicity and, hence, there is no reason to believe that economic variables should

present the same lead/lag cross-correlation at all frequencies. We incorporate this rationale into the context of the single-factor C-CAPM by using standard techniques to estimate the coherency (the analog of the correlation coefficient in the time domain) and the gain (the analog of the regression coefficient) between consumption risk over the frequency domain and returns. Measuring the portfolio risk of consumption over the whole frequency domain enables us to separate different layers of dynamic behavior within the standard C-CAPM by distinguishing between the short run (fluctuations of 2 to 6 quarters), the medium run or business cycle (lasting from 8 to 32 quarters), and the long run (oscillations of duration above 32 quarters). If consumption risk is a more persistent process than suggested by the conventional analysis, identifying the impact of lower frequencies of consumption risk can alter the implied long-run riskiness in ways that are empirically important and cannot be addressed by standard time-domain techniques, which aggregate over the entire frequency band and are not robust when frequency variations are large.\(^2\)

To attain our goal we modify the standard two-step Fama and French (1992) estimation procedure typically employed in the literature, which involves regressing the portfolio return on consumption growth and using the estimated coefficients in a cross-section regression to estimate the components of the C-CAPM (price of risk, equity premium, risk aversion), in the following way. We employ a spectral decomposition of the series at hand and we then use these estimates in the cross-section equation to obtain the equity premium, the price of risk, as well as the implied measure of ‘pseudo’ risk aversion for each frequency.\(^3\) Our approach can thus circumvent several caveats associated with unmodeled frictions, time aggregation or measurement error in consumption data, which are often found to account for the short-run predictability of the pricing errors.

Our findings indicate that at high frequencies of consumption risk the evidence coincides with those reported by the existing literature: consumption risk does not explain satisfactorily the variation in returns. However, when lower frequencies of consumption risk are examined and thus the horizon of consumption growth increases (eventually reaching infinity) consumption risk can explain up to 98% of the cross-sectional variation of expected returns and the equity

\(^2\)For example, employing a standard VAR model between 2 variables and \(k\) lags requires the solution of a \(2k\) eigenproblem for both eigenvalues and eigenvectors to assess the relative importance of each cyclical component. More importantly, the limiting covariance structure as the horizon tends to infinity cannot be estimated.

\(^3\)See sections 2 and 3 for more details on this procedure.
premium puzzle is eliminated. These findings are robust to the definitions of the variables, the sample span and the set of portfolios utilized. Moreover, given the importance of long-run consumption risk for the dynamics of the C-CAPM, we address the impact of long-term risk-free rates within this spectral approach and we find that the model preserves its significance for low frequencies of consumption risk.

We are thus able to provide additional insights into the relationship between returns and long-term consumption dynamics by confirming that consumption risk can provide useful information for the variation of excess returns when examined over the frequency domain. In this respect, we further highlight the importance of long-run consumption risk by explaining a larger share of cross-sectional variation of expected returns. It is worth noting that the spectral estimation of consumption-based models has also been considered by Berkowitz (2001) and Cogley (2001). Berkowitz (2001) has proposed a one-step Generalized Spectral estimation technique for estimating parameters of a wide class of dynamic rational expectations models in the frequency domain. By applying his method to the C-CAPM he finds that when the focus is oriented towards lower frequencies, risk aversion attains more plausible values at the cost of a risk-free rate puzzle generated by low estimates of the discount factor. Cogley (2001) decomposes approximation errors over the frequency domain from a variety of stochastic discount factor models and finds that their fit improves at low frequencies, but only for high degrees of calibrated risk aversion. In our paper we show how the spectral analysis can be incorporated in the aforementioned standard two-step estimation methodology in an easily implementable way to address satisfactorily the equity premium puzzle and the cross-sectional variation when low-frequencies of consumption risk are utilized.

We close the introduction by noting that the paper is part of the upcoming literature that aims at capturing the behavior of aggregate and cross-sectional stock returns via the long-term dynamics of consumption. The approach adopted here remains, however, agnostic about the driving force of these dynamics. For instance, our findings are consistent with the general class of models that relax the assumption of costless adjustment in consumption plans by including the time spent to calculate and implement a new consumption-savings decision, or constraints in information and search costs that lead investors in making infrequent consumption and portfolio allocation decisions at discrete points in time. The impact of consumption risk measured over
the frequency domain can also be consistent with models that entail monitoring costs and heterogeneous agents, in which only a fraction of households adjusts its consumption over discrete intervals.\footnote{See, for instance, Grossman and Laroque (1990), Lynch (1996), Marshall and Parekh (1999), Gabaix and Laibson (2001) and Jagganathan and Wang (2005)}

The rest of the paper is organized as follows. Section 2 presents long-term consumption risk within the C-CAPM and its modified version in the context of spectral analysis. Section 3 presents the data and the estimation methodology. Section 4 presents the empirical results for consumption risk over the frequency domain and section 5 presents some robustness tests. Section 6 investigates the impact of long-term risk-free rates and section 7 concludes the paper.

2. Expected returns and the risk to consumption over the frequency domain

The standard C-CAPM assumes that the representative household maximizes the expected present discounted value of utility flows from consumption by allocating wealth to consumption and different investment opportunities. At the optimal allocation a marginal investment at time $t$ in any asset should yield the same expected marginal increase in utility at $t + 1$, which for the constant relative risk aversion utility function implies that:

$$E_t[C_{t+1}^{-\gamma} R_{j,t+1}] = E_t[C_{t+1}^{-\gamma}] R_{t,t+1}^j$$

where $C_{t+1}$ is consumption at $t + 1$, $R_{j,t+1}$ is the gross real return on portfolio $j$ of stocks unknown at $t$ and known at $t + 1$, $R_{t,t+1}^f$ is the gross real return on a risk-free asset between $t$ and $t + 1$, and $\gamma$ is the representative household’s constant coefficient of relative risk aversion. Equation (1) can be written as a model of average cross sectional returns by manipulating it to a beta representation or factor model, in which the expectation of the equity premium, $E[R_{j,t+1}^e] = E[R_{j,t+1} - R_{t,t+1}^f]$, is given in terms of covariances by:

$$E[R_{j,t+1}^e] = \alpha_0 + \beta_{j,0} \lambda_0$$

where $\alpha_0 = 0$, $\beta_{j,0} = \frac{Cov[\Delta \ln C_{t+1}, R_{j,t+1}^e]}{Var[\Delta \ln C_{t+1}]}$, $\lambda_0 = \frac{\gamma Var[\Delta \ln C_{t+1}]}{\left[1 - \gamma \Delta \ln C_{t+1}\right]}$. Equation (2) provides an external test of the structure embodied in the model with consumption growth, $\Delta \ln C_{t+1}$, being the stochastic discount factor that prices returns. The estimated $\alpha_0$ should be equal to zero and the expected excess return on a portfolio is equal to the scaled consumption risk of the portfolio,
The estimated $\lambda_0$ and moments of consumption growth imply a level of the risk aversion for the representative investor according to:

$$\gamma = \frac{\lambda_0}{E[\Delta \ln C_{t+1}]\lambda_0 + Var[\Delta \ln C_{t+1}]}$$

Equations (1) to (2) evaluate the risk of a portfolio based solely on its covariance with contemporaneous consumption growth. They maintain the assumption that the intertemporal allocation of consumption is optimal from the perspective of the textbook model of consumption smoothing, so that any change in marginal utility is reflected instantly and completely in consumption.

Parker (2001, 2003) and Parker and Julliard (2005) have allowed for the slow response of consumption to market returns and have evaluated the risk/return trade-off among portfolios of stocks by focusing on the ultimate consumption risk measured by the covariance of the return at $t + 1$ and the change in consumption from $t$ to $t + 1 + s$, where $s$ is the horizon over which the consumption response is studied:

$$Cov[\ln \left(\frac{C_{t+1+s}}{C_t}\right), R_{j,t+1}^e]$$

In terms of beta representation we have:

$$E[R_{j,t+1}^e] = \alpha_s + \beta_{j,s}\lambda_s$$

where $\alpha_s = 0$, $\beta_{j,s} = \frac{Cov[\ln(\frac{C_{t+1+s}}{C_t}), R_{j,t+1}^e]}{Var[\ln(\frac{C_{t+1+s}}{C_t})]}$, $\lambda_s = \frac{\gamma_s Var[\ln(\frac{C_{t+1+s}}{C_t})]}{E[1-\gamma_s \ln(\frac{C_{t+1+s}}{C_t})]}$. When $S = 0$, equation (5) yields the standard beta representation (2). For $S > 0$, the stochastic discount factor considered is one minus the long-horizon consumption growth times the risk aversion of the representative agent, $\gamma_s$. The estimated $\lambda_s$ and moments of consumption growth imply then a level of relative risk aversion given by:

$$\gamma_s = \frac{\lambda_s}{E[\ln(\frac{C_{t+1+s}}{C_t})]\lambda_s + Var[\ln(\frac{C_{t+1+s}}{C_t})]}$$

Equations (5) and (6) show a modification of the standard C-CAPM over the time domain. In their empirical results, Parker and Julliard (2005) find a model improvement as the horizon
increases accompanied by lower estimates of the risk-free rate and the coefficient of risk aversion, but do not report results beyond 15 quarters as the trade-off between a larger horizon and optimal inference leads to a choice of 11 quarters as the preferred specification.

Clearly by varying the horizon, $S$, consumption risk takes a range of values from the short-run to the long-run along with the corresponding asset pricing implications of these risks. Defining $\ln\left(C_{t+1+s}/C_t\right) \equiv \Delta^s \ln C_t$ from $\Delta \ln C_t$ through the transformation $H(L) = (1 + L + L^2 + \ldots + L^s)$, where $L$ is the lag operator. The spectrum of $\Delta^s \ln C_t$, $f_{\Delta^s \ln C_t}$, is then linked with the one of $\Delta \ln C_t$ by $f_{\Delta^s \ln C_t} = H(e^{-i\omega})H(e^{i\omega})f_{\Delta \ln C_t}$, where the frequency $\omega$ is a real variable in the range $0 \leq \omega \leq \pi$; for example, for $s = 2$, $f_{\Delta^2 \ln C_t} = (2 + 2 \cos \omega)f_{\Delta \ln C_t}$. For $\omega = 0$, the variance of $\Delta^2 \ln C_t$ is 4 times the variance of $\Delta \ln C_t$, while the respective variance for $\omega = \pi$ is eliminated. This transformation strengthens lower frequencies (long-run) and attenuates the impacts of the higher ones (short-run).

Now, we can measure consumption growth at any frequency $\omega$ by assuming, for instance, that agents care about risk associated with fluctuations of consumption at some frequencies differently from other frequencies, and in turn calculate a measure of consumption risk over the frequency domain given the covariance between consumption growth at a given frequency, $\Delta \ln C_t(\omega)$, and the excess return at any frequency, $R_{jt}^e$. In this respect, after dropping the time subscript for notational simplicity, the beta-form representation given in (2) and (5) can be modified to its frequency domain counterpart that yields the response of excess returns to consumption risk over the whole band of frequencies, $\omega$, as follows:

$$E[R_j^e] = \alpha_\omega + \beta_{j,\omega} \lambda_\omega$$

(7)

where $\alpha_\omega = 0$ and the beta coefficient, $\beta_{j,\omega}$, is the gain between returns and consumption growth denoted by $G_{R_j^e, \Delta \ln C}(\omega)$ and defined as the ratio of the cross-spectrum of the series at hand over the spectrum of consumption growth at a given frequency:

$$G_{R_j^e, \Delta \ln C}(\omega) \equiv \frac{|f_{R_j^e, \Delta \ln C}(\omega)|}{f_{\Delta \ln C, \Delta \ln C}(\omega)}$$

(8)

The gain provides us with a scalar measure of the amplitude of the relationship between the
\(\omega\)-frequency component of \(R^j_t\) on the corresponding component of \(\Delta \ln C\) since it is a fact of cross-spectral densities that the covariance between consumption growth in a given frequency and returns at any frequency is the same as the covariance between consumption growth at a given frequency and returns on the same frequency.

Once the price of risk, \(\lambda_\omega\), is estimated from the cross-section regression (7), we can calculate the implied coefficient of ‘pseudo’ relative risk aversion of the representative agent at each frequency as:

\[
\gamma_\omega = \frac{\lambda_\omega}{E[\Delta \ln C|\lambda_\omega + \int \Delta \ln C, \Delta \ln C(\omega)]}
\]  

(9)

We refer to \(\gamma_\omega\) as ‘pseudo’ relative risk aversion because the integral of \(\gamma_\omega\) over the whole band of frequencies is not equal to its time domain counterpart due to the non-additiveness property of the above formula.\(^5\) Furthermore, the calculation of \(\gamma_\omega\) involves the mean of consumption growth over the time domain in the denominator while the remaining elements of the formula are related to a specific frequency. Keeping in mind these caveats, we report the value of ‘pseudo’ risk aversion for each specification in order to understand the model properties over various frequencies.

3. Data and estimation methodology

3.1. Data and spectral properties

For our portfolios and returns series we use quarterly returns on the 25 Fama and French portfolios, which are the intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the ratio of book equity to market equity (B/M). B/M used during a fiscal year is based on the book equity for the previous fiscal year divided by ME for December of the previous year. The B/M breakpoints are the NYSE quintiles. The portfolios include all NYSE, AMEX, and NASDAQ stocks for which there is market equity data for December and June of the previous fiscal year, and (positive) book equity data for the previous fiscal year. The series are available on a monthly basis and excess returns are constructed by subtracting the three-month Treasury Bill rate, which proxies the risk-free rate. To match consumption data we use a quarterly frequency and set our timing convention so that \(R_{j,t+1}\) represents the return on portfolio \(j\) during the quarter \(t+1\). We measure consumption as personal consumption.

\(^5\)The definition of preferences over frequency components of a stochastic process is not as straightforward as it might seem; see Otrok (2001) on spectral welfare cost functions.
expenditures on nondurable goods from the National Income and Product Accounts. We make
the ‘end-of-period’ timing assumption that consumption during quarter $t$ takes place at the
end of the quarter. The data are made real using a chain weighted price deflator, spliced
across periods, produced by the Bureau of Economic Analysis. These series determine the
sample, which covers the second quarter of 1947 to the last quarter of 2001, and the frequency
(quarterly) utilized.\footnote{We obtained the Fama and French portfolio data from Kenneth French’
's web page (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The rest of
the data were obtained from Jonathan Parker’s web page (http://www.princeton.edu/~jparker/
research/crisk.html); see Parker and Julliard (2005) for a more detailed description of the dataset.}

Before moving on with the estimation methodology, we report some evidence on the co-
movement between returns and consumption growth in the frequency domain. The spectra and
cospectra of a vector of time-series for a sample of $T$ observations can be estimated for a set of
frequencies $\omega_n = 2\pi n/T$, $n = 1, 2, \ldots, T/2$. The relevant quantities are estimated through
the periodogram, which is based on a representation of the observed time-series as a superposition
of sinusoidal waves of various frequencies; a frequency of $\pi$ corresponds to a time period of two
quarters, while a zero frequency corresponds to infinity. However, the estimated periodogram
is an unbiased but inconsistent estimator of the spectrum because the number of parameters
estimated increases at the same rate as the sample size. Consistent estimates of the spectral
matrix can be obtained by either smoothing the periodogram, or by employing a lag window
approach that both weighs and limits the autocovariances and cross-covariances used.\footnote{For
example, the spectrum of $x_t$ is estimated by $f_{xx}(\omega) = \frac{1}{T} \sum_{k=-(T-1)}^{T-1} w(k) \hat{\rho}_k e^{-i\omega k}$, where the kernel, $w(k)$, is a series of lag windows.}

We use here the Bartlett window that assigns linearly decreasing weights to the autocovariances
and coscovariances in the neighborhood of the frequencies considered and zero weight thereafter.\footnote{The Bartlett window has the following form: $\lambda(s) = \begin{cases} 1 - \frac{|s|}{k}, & |s| \leq k \\ 0, & |s| > k \end{cases}$ while the bandwidth, $k$, is set using the rule $k = 2\sqrt{T}$, as suggested by Chatfield (1989), where $T$ is the sample size.}

Figures 1A and 1B depict the spectra of the series under scrutiny (along with 95% confidence
intervals) and can be interpreted as the variance decompositions over various frequency bands
(stated as a fraction of $\pi$).\footnote{The series employed are demeaned before any spectral measure is estimated, as spectral analysis pertains to stationary zero-mean processes.} As can be readily observed, the variability of excess returns
does not exhibit substantial changes over the frequency domain.\footnote{Confidence intervals were derived based on a normal approximation of the spectra of the series; see Priestley (1981) for a detailed description.} On the other hand, the
variability of non-durables consumption is muted for 2 to 32 quarters; however, for horizons exceeding 32 quarters a steep increase is prevalent. As $t$ approaches infinity, the variance of consumption is seven times greater than its 32-quarter value and 52 times greater than its short-run value. The concentration of variance in low frequencies is an indication of short-term correlation in consumption growth, such as an AR(1) process with a positive coefficient, rather than an indication of non-stationarity of the process, which can be ruled out for the series at hand.\footnote{See Campbell (2003, section 3.2) and the references cited therein for some evidence the properties of US consumption growth.} This finding has direct implications for the subsequent analysis, especially when the coefficient of 'pseudo' risk aversion is calculated from the estimates of our model, because the variance of consumption growth is inversely related to the coefficient of risk aversion by (3) and (9) and hence we expect that ‘pseudo’ risk aversion will decrease as the lower frequencies are taken into account.

For expositional purposes, we also employ as a measure of comovement between returns and consumption risk over the frequency domain, the well-known squared coherency, $c^2_{R^e_j,\Delta \ln C}(\omega)$, defined here as:

$$c^2_{R^e_j,\Delta \ln C}(\omega) \equiv \left| \frac{f_{R^e_j,\Delta \ln C}(\omega)}{f_{\Delta \ln C,\Delta \ln C}(\omega)f_{R^e_j,R^e_j}(\omega)} \right|^2 = \frac{C^2_{R^e_j,\Delta \ln C} + Q^2_{R^e_j,\Delta \ln C}}{f_{\Delta \ln C,\Delta \ln C}(\omega)f_{R^e_j,R^e_j}(\omega)} \tag{10}$$

where $0 \leq c_{R^e_j,\Delta \ln C}(\omega) \leq 1$, $f_{R^e_j,\Delta \ln C}(\omega) = C_{R^e_j,\Delta \ln C}(\omega) - iQ_{R^e_j,\Delta \ln C}(\omega)$ is the cross-spectrum between any two variables, which is complex-valued and therefore can be decomposed into its real and imaginary components, $C_{R^e_j,\Delta \ln C}(\omega)$ termed the co-spectrum, and $Q_{R^e_j,\Delta \ln C}(\omega)$ termed the quadrature spectrum, respectively.\footnote{See Hamilton (1994) for a general overview of spectral analysis.} Intuitively, coherency provides a measure of the correlation between the market excess return and non-durables consumption growth at each frequency and can be interpreted as the frequency domain analog of the correlation coefficient.

Figure 1C presents the coherency (along with 95\% confidence intervals) between the two series. This analysis has been undertaken for every portfolio but to save space we report only the results for the aggregate market return. Overall our estimates suggest that the correlation (measured by coherency) between returns and consumption growth exhibits an upward trend as we move from high to low frequencies. Specifically, as regards the short-run correlation...
for frequencies between $\pi$ and $7\pi/8$ corresponding to around 2 quarters, coherency fluctuates around 20%. Then it plunges to around 5% and steadily increases to reach a local peak of 60% at frequencies corresponding to 3-4 quarters. Two more cycles are observable with peaks at 6 and 16 quarters. The maximum is reached at zero frequency, i.e. for an infinite horizon. In this case, the coherency between the series at hand is estimated at 79%. On the whole, the short-run correlation between returns and consumption growth is low, the business-cycle correlation amounts on average to roughly 50%, while the long-run correlation exceeds 70%.

3.2. Estimation methodology

Estimation of (2) is typically performed in the literature within a two-step approach (Fama and French, 1992). The first step involves a time series regression of the return of the $j$ portfolio, $R_{e,j,t+1}$, onto a constant and consumption growth, $\Delta \ln C_{t+1}$, in order to obtain an estimate of the slope coefficient $\beta_{j,0}$. As a second step, the estimated coefficients are employed in the cross-section regression (2) in order to get the estimate of the price of risk, $\lambda_0$. By employing excess returns, we can test whether our model contains an equity premium by simply testing the significance of the constant. The adjusted $R^2$ of this equation measures the fraction of the cross-sectional variation explained by the data. Furthermore, inference regarding the risk aversion of the representative investor can be conducted taking as given the mean and variance of consumption growth by employing (3) with the standard errors of $\gamma$ calculated by the delta method.

Our methodology differs from the one described above only in the first step, i.e. in the way betas are obtained. Specifically, employing the spectral decomposition of the series outlined in subsection 3.2., we calculate the gain between each portfolio’s excess return and consumption growth for each frequency as the ratio between the co-spectra of the series and the spectrum of consumption growth as given by formula (8). The estimated gains / betas, $\beta_{j,\omega}$, for each portfolio $j$, are then employed as regressors in the following cross-section equation

$$ E[R_j^e] = \hat{\alpha}_\omega + \beta_{j,\omega} \hat{\lambda}_\omega $$

which yields a testable form of the C-CAPM in the frequency domain. Our handling of this equation is the same as for any cross-sectional regression. We obtain the estimates $\hat{\alpha}_\omega, \hat{\lambda}_\omega$,

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14 Alternatively, the Fama and MacBeth (1973) methodology can be employed.
by means of ordinary least squares estimation correcting for possible heteroscedasticity by employing Newey-West standard errors. The adjusted $R^2$ of the equation gives as a measure of success of the frequency domain C-CAPM to explain the cross-sectional variation of returns. The statistical significance of the intercept, $\widehat{\alpha}$, is directly related to the existence of an equity premium. Furthermore, the estimate of the price of risk, $\widehat{\lambda}$, should be significant and positive. This estimate can then be fed into (9) in order for the coefficient of ‘pseudo’ relative risk aversion to be calculated along with its standard error which is obtained by means of the delta method. As already mentioned, this coefficient should be interpreted with caution as it does not constitute a spectral decomposition of its time domain counterpart since the additiveness property is not preserved.

4. Empirical findings

This section asks whether consumption risk explains the cross-sectional variation in expected returns for various frequencies. In particular, the questions we seek to answer are the following. First, does consumption risk at various frequencies explain a large share of variation of average returns? Second, is the price of risk, $\lambda$, statistically significant? Third, does the estimate of $\alpha$ corroborate the existence of an equity premium? Last, what is the estimate of the ‘pseudo’ risk aversion coefficient, $\gamma$?

To allow for comparisons with the rest of the literature, in this section we take the standard route and we estimate the model by employing non-durables consumption and gross excess returns from the 25 Fama-French portfolios. As a first step, we estimate model (7) by imposing the coefficient restriction $\alpha = 0$. Table 1 (Panel A) reports the estimation results for a range of frequencies corresponding from 2 quarters to infinity. The first row reports the evidence for the highest frequency considered (which corresponds to two quarters in the time domain). Our results suggest that at this frequency consumption risk does not explain the variation in returns and is associated with a significant and positive price of risk (given by the estimate of $\lambda$). The coefficient of ‘pseudo’ risk aversion is estimated at 71 and is found to be significant.

These findings are in line with those typically reported in the literature on the C-CAPM. As we move to lower frequencies (and consequently increase the time horizon) consumption risk still fails to explain a larger share of the cross-sectional variation; however, the implied ‘pseudo’ risk aversion declines almost monotonically and reaches 8.8 for the 16-quarter horizon. When even
lower frequencies are taken into account the performance of the model improves substantially. For the 32-quarter horizon, consumption risk is positive and significant and explains 66% of the cross-sectional variation of the returns. The coefficient of ‘pseudo’ risk aversion is significant and reduced to 4.6. The performance of the C-CAPM is further improved at zero frequency (infinite horizon). The model succeeds in explaining 98.1% of the cross-sectional variation of returns. The associated price of risk is significant and estimated at 0.007 (0.677x10^{-2}), almost three times greater than, for example the one at 2-quarters, whereas ‘pseudo’ risk aversion is estimated at just 4.3 and remains significant.

Next, we assess model (7) by estimating $\alpha_\omega$ rather than imposing $\alpha_\omega = 0$. In this respect, we separately evaluate the ability of the model to explain the equity premium and the cross section of expected stock returns, and we are able to measure the extent to which the model addresses the equity premium puzzle. Panel B of Table 1 reports the estimation results. The evidence suggests that at a high frequency consumption risk does not explain variation in returns and is associated with a significant equity premium of the magnitude of 2.3% per quarter. The coefficient of ‘pseudo’ risk aversion is estimated at 42.5 and found to be insignificant. This poor performance of contemporaneous consumption risk is also depicted in the upper left panel of Figure 2, which plots the consumption betas (gains) and the average realized returns along with the second-stage regression line associated with this frequency. The overall picture indicates an almost flat relationship between consumption risk and returns at this frequency. Figure 3 plots in turn the predicted and average returns of the portfolios. The horizontal distance between a portfolio and the 45-degree line is the extent to which the expected return based on fitted consumption risk (on the vertical axis) differs from the observed average return (on the horizontal axis). As expected, at the 2-quarter horizon there is almost no relation between predicted and realized returns.

When we move to lower frequencies consumption risk explains a larger share of the cross-sectional variation, reaching 12% for the 8-quarter horizon. However, the implied premium remains large and significant, whereas the price of risk turns out insignificant and negative. This general picture is also depicted in the regression line in the upper right part of Figure 3. Furthermore, a significant and high ‘pseudo’ risk aversion is estimated at this frequency reaching 20.9. Similar findings pertain with respect to the 16-quarter frequency with ‘pseudo’
risk aversion now declining and reaching 6.8, but with a large standard error.

As lower frequencies are further considered the performance of the model improves substantially. For the 32-quarter horizon, consumption risk is positive and significant, and explains 66% of the cross-sectional variation of the returns. These findings are depicted in the lower left panel of Figures 2 and 3. The regression line is positive, quite steep and suggests a strong relationship between betas and returns. As expected, the deviation between fitted and realized returns is sufficiently reduced. The coefficient of ‘pseudo’ risk aversion becomes significant and is now reduced to 4.6. Associated with this horizon is a negligible and insignificant equity premium of -0.3%. The performance of the C-CAPM is further improved at zero frequency (infinite horizon). The model succeeds in explaining 98.6% of the cross-sectional variation of returns coupled with an insignificant pricing error. The associated price of risk is significant and estimated at 0.007; however, at this frequency our model overpredicts average returns by just 0.2%, which is marginally significant, whereas ‘pseudo’ risk aversion is estimated at 4.3 and is significant. These features are also illustrated in the lower right part of Figures 2 and 3, in which the average realized and fitted returns are almost perfectly aligned on the regression line and the 45-degree line, respectively.

To sum up, we find that when higher frequencies of consumption risk are considered the results replicate the typical findings of the literature, i.e. the C-CAPM fails to explain the differences in expected stock returns by the variation in the covariance of consumption and returns. In contrast, as lower frequencies of consumption risk are taken into account, consumption risk explains almost entirely the cross-sectional variation of expected returns and the equity premium puzzle is eliminated. Finally, although the coefficient of ‘pseudo’ risk aversion reported here is not directly comparable to the estimates of risk aversion typically reported in the relevant literature, its implied values by the use of long-run consumption risk of stockholders are as low as 3.3 to 4.3. This range of values, which is far below the level of 10 considered as reasonable by Mehra and Prescott (1985), stems mainly from the increased variability of long-run consumption that is inversely related to risk aversion.

5. Robustness tests

In this section we present some sensitivity tests on the relationship between consumption risk and the expected returns over the frequency domain. We first consider the impact of
alternative specifications by using a smaller sample size as well as alternative definitions of returns and consumption, and subsequently we examine the impact of alternative portfolios on our results.

5.1. Alternative specifications

Some studies (including, among others, Fama and French, 1992, 1993, and Lettau and Ludvigson, 2001) have used a shorter time period than the one analyzed in our baseline results. To allow for comparisons, Panel A of Table 2 shows the results of estimating our model on a sample of returns that starts in the third quarter of 1963. In this sub-period, the pattern of coefficients and the fit tell a similar story, except that low-frequency consumption risk does even better at explaining expected returns. Around 67% and almost 100% of the variation in expected returns is explained by consumption risk over the 32-quarter and infinite horizons with the level of ‘pseudo’ risk aversion again found to be slightly larger than 4 (reaching 4.6 and 4.3, respectively). Similar to the baseline specification, the fitted model understates the average return on all portfolios by 0.5% and 0.2%. The fit of the model for the infinite horizon is depicted on the upper left part of Figure 4.

Second, we measure consumption risk using total consumption instead of non-durables consumption. Ait-Sahalia et al. (2004) have argued that the consumption risk of equity is understated by NIPA nondurable goods because it contains many necessities and few luxury goods. As pointed out by the authors, consumers have more discretion over their consumption of luxury goods than essential goods, and consumption of the former is found to covary more strongly with stock returns.\textsuperscript{15} Panel B of Table 2 shows that using total consumption risk in place of nondurables consumption risk leads to a slightly different picture. Long-run total consumption risk fits the cross-section of expected returns somewhat better than non-durables consumption and, interestingly, lowers the level of ‘pseudo’ risk aversion relative to the previous specifications at 3.3. This finding accords well with nonseparability over time (or habits) in the utility function, which is expected to be stronger for durable consumption goods now included in consumption. Past consumption levels are expected to affect more negatively the marginal utility of consumption for durable goods when longer horizons are considered, which drives down the

\textsuperscript{15}See also Parker (2001). The usual concern when total consumption is used is that it contains the flow of expenditures on durable goods instead of the theoretically desired- stock of durable goods. However, expenditures and stocks are cointegrated and, hence, the long-term movement in expenditures following an innovation to equity returns also measures the long-term movement in consumption flows.
estimates of ‘pseudo’ risk aversion. The upper right part of Figure 4 plots the performance of the specification with total consumption.

Finally, we use consumption risk over the frequency domain to price long-horizon returns. Long-horizon returns are calculated as cumulative returns over the next 11 quarters.\(^\text{16}\) Panel C of Table 2 shows some improvements of our model for shorter horizons compared to the baseline specification. Specifically, for an 8-quarter horizon, the model succeeds in explaining almost half the cross-sectional variation of returns; however, the price of risk is negative and the associated ‘pseudo’ risk aversion is found to be quite high, estimated at 18.8. As we move to lower frequencies, and specifically to the 32-quarter horizon the explanatory power of the model is lower than the baseline specification (42.2\% as opposed to 65.5\%), but the remaining attributes of the model are in line with the theoretical ones. The price of risk is positive and significant, the equity premium is insignificant and the estimated ‘pseudo’ risk aversion decreases to 4.7. This specification yields similar findings to the baseline specification for the infinite horizon and its performance is depicted at the bottom part of Figure 4.

5.2. Other portfolios

The C-CAPM as any asset pricing model should be able to explain expected returns on any set of portfolios. So far, the portfolios considered are the double-sorted 25 Fama-French B/M and ME value-weighted portfolios, which basically aim at capturing the value and size premia. We consider here alternative portfolios sorted on both firm characteristics and overall economic factors or systematic risk factors in order to check whether consumption risk over the frequency domain succeeds in explaining risk premia generated by these portfolios.

As a first step, we consider a slightly different set of returns, namely the 25 equal-weighted Fama-French portfolios that are also examined by Parker and Julliard (2005). In line with these authors, low-frequency consumption risk does an even better job of explaining the cross-sectional pattern of expected returns for these portfolios (see Panel A of Table 3). A slightly increased proportion of the variation in expected returns is explained along with low coefficients of ‘pseudo’ risk aversion, whereas the equity risk premium is found to be insignificant. The fit of the model for the infinite horizon is depicted on the upper left part of Figure 5.

Second, we consider a set of single sorted portfolios, namely the 10 size (ME), 10 book to

\(^{16}\)For comparison purposes, the choice of the horizon is the one that corresponds to the selected model of Parker and Julliard (2005).
market (B/M) and 10 dividend yield (D/P) portfolios of Fama and French. These portfolios sort firms on the basis of their characteristics that lead to cross-sectional dispersion in measured risk premia and are behind the factor models of Fama and French (1993).\textsuperscript{17} This set of portfolios aims at disentangling the value and size premia. To the extent that the C-CAPM holds, we expect to find growth firms to have less exposure to consumption risk than value firms and smaller firms to be exposed to higher consumption risk when compared to larger firms; see also Jagannathan and Wang (2005) and Cochrane (2005). Our results (reported in Panel B of Table 3) are in line with those of our baseline specification. At a high frequency, the C-CAPM explains 13% of the cross-sectional variation in expected returns associated with a significant risk premium and a high coefficient of ‘pseudo’ risk aversion estimated at 57.7. The fit of the model improves, whereas the estimate of risk aversion decreases with the frequency decline. At the 32-quarter horizon, half of the variation is explained and ‘pseudo’ risk aversion declines to 4.6, while at an infinite horizon, the respective figures are 93.9% and 4.3. The upper right part of Figure 5 plots the actual and the predicted returns for this set of portfolios.

Third, we use the 20 risk-sorted portfolios employed by Campbell and Vuolteenaho (2004).\textsuperscript{18} The authors follow Daniel and Titman’s (1997) point that sorting only on firm characteristics could generate a spurious link between premia and risk measures, and sort common stocks into 20 portfolios according to their past loadings with state variables that are useful in predicting the aggregate market return.\textsuperscript{19} The purpose of their strategy is to generate portfolios with a large spread in these loadings and thus overcome Daniel and Titman’s (1997) problem. Panel C of Table 3 reports our results for this set of portfolios. Interestingly, the C-CAPM fails in at least one of its aspects for all the frequencies under consideration with the exception of the infinite

\textsuperscript{17}The 10 size value-weighted portfolios are formed on the basis of market capitalization and include all NYSE, AMEX, and NASDAQ stocks in the CRSP database which are ranked at the end of June of each year using NYSE capitalization breakpoints. The 10 B/M portfolios are formed at the end of each June using NYSE breakpoints. The BE used in June of year $t$ is the book equity for the last fiscal year ending in $t-1$ and ME is price times shares outstanding at the end of December of $t-1$. The 10 D/P portfolios include all NYSE, AMEX, and NASDAQ stocks for which ME for June of year $t$, and at least 7 monthly returns (to compute the dividend yield) from July of $t-1$ to June of $t$ are available. Portfolios are formed on D/P at the end of each June using NYSE breakpoints. The dividend yield used to form portfolios in June of year $t$ is the total dividends paid from July of $t-1$ to June of $t$ per dollar of equity in June of $t$. The returns on these portfolios are taken from Kenneth French’s web site, where more details on their construction can be found.

\textsuperscript{18}These portfolios are available at http://post.economics.harvard.edu/faculty/vuolteenaho/papers.html.

\textsuperscript{19}These state variables include the excess log return on the market, the term yield spread (computed as the difference between ten-year and short-term bonds) and the small stock-value spread (computed as the difference between the log(B/M) of the small high B/M portfolio and the small low B/M portfolio). More details can be found in the Appendix of Campbell and Vuolteenaho (2004), which is available at http://kuznets.fas.harvard.edu/~campbell/papers.html.
horizon. For this horizon, 82% of the cross-sectional variation of the returns is explained and ‘pseudo’ risk aversion is estimated at 4.3. Figure 5 (bottom part), which plots realized returns versus predicted returns, shows that the spread in returns across portfolios is lower than the one generated by the portfolios considered so far explaining the somewhat worse performance of this model.

Fourth, we consider 34 industry-sorted portfolios, which have posed a particularly challenging feature from the perspective of systematic risk measurement (see Fama and French, 1997). Value-weighted industry portfolios are formed by sorting all NYSE, AMEX, and NASDAQ stocks by their CRSP four-digit SIC Code at the end of June of each year. Similar to the previous set of portfolios, our findings suggest that systematic industry-specific risk is priced only for the infinite horizon (see Panel D of Table 3). The coefficient of ‘pseudo’ risk aversion for the 32-quarter and the infinite horizon is estimated at 4.2, a value that is very close to the one attained by every specification and portfolio considered when non-durables consumption is employed.

6. Long-run risk-free rates and consumption risk over the frequency domain

The previous sections have established that the low frequencies of consumption risk, which are associated with the long-run pattern of C-CAPM, improve the empirical fit of the model. An extension of this approach envisages the impact of risk-free rates of longer maturity, which are likely to embed useful information when the horizon of consumption risk widens. Intuitively, if long-term interest rates are negatively related to consumption growth, then they provide a hedge against bad states and individuals will sell short-term bonds and buy long-term bonds to receive payoffs when their consumption level is expected to be lower, thus resulting in a falling or negative term structure. On the flip side, if long-run rates earn a low return when consumption growth is negative, holding long-term bonds exacerbates consumption risk resulting in a rising term premium.

To assess the impact of long-run risk-free rates and consumption risk over the frequency

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20 The industry definitions are available at Kenneth French’s web site. We include in our analysis the portfolios for which we have returns for the whole sample period.

21 Estrella and Mishkin (1996) have found that inverted yield curves can be leading indicators of recessions and hence of reduced consumption growth rates. The empirical implications of long-run risk-free rates (and the associated term structure) for the C-CAPM have been investigated by several studies including, among others, Harvey (1988, 1989, 1991, 1993), Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994), Kamara (1997), Roma and Torous (1997), and Hamilton and Kim (2002). The general empirical contention from these studies is that the slope of the term spread is positively associated with future economic activity.
domain, we follow Parker and Juliard (2003) and develop in the next section a variant of the model presented in section 2 that incorporates risk-free rates of longer maturity in the C-CAPM and then we present some empirical results.

6.1. A beta representation of long-run risk-free rates and consumption risk

The solution of the investor optimization problem implies that:

\[
E_{t+1}\left[ \frac{1}{1 + \rho} (1 + R_{t+1,t+1+s}^f) \frac{u'(C_{t+1+s})}{u'(C_{t+1})} \right] = 1
\]

(11)

where \( \rho \) is the rate of time preference and \( R_{t+1,t+1+s}^f \) is the risk-free rate with \( s \)-periods ahead maturity. In turn, we can re-write the Euler equation (1) as:

\[
E_{t}[R_{t+1,t+1+s}^f \frac{u'(C_{t+1+s})}{u'(C_t)} R_{j,t+1}] = E_{t}[R_{t+1,t+1+s}^f \frac{u'(C_{t+1+s})}{u'(C_t)}] R_{j,t+1}^f
\]

(12)

Assuming that \( R_{t+1,t+1+s}^f \) is orthogonal to \( R_{j,t+1}^f \), we can get the following beta representation for the excess return of portfolio \( j \):

\[
E[R_{j,t+1}^e] = \alpha^s + \beta_j^s \lambda^s
\]

(13)

where

\[
\alpha^s = \frac{Cov[R_{t+1,t+1+s}^f \frac{u'(C_{t+1+s})}{u'(C_t)}, R_{t+1,t+1+s}^f]}{E[R_{t+1,t+1+s}^f \frac{u'(C_{t+1+s})}{u'(C_t)}]}, \quad \beta_j^s = \frac{Cov[R_{t+1,t+1+s}^f \frac{u'(C_{t+1+s})}{u'(C_t)}, R_{j,t+1}^e]}{Var[R_{t+1,t+1+s}^f \frac{u'(C_{t+1+s})}{u'(C_t)}]}
\]

and \( \lambda^s = -\frac{Var[R_{t+1,t+1+s}^f \frac{u'(C_{t+1+s})}{u'(C_t)}]}{E[R_{t+1,t+1+s}^f \frac{u'(C_{t+1+s})}{u'(C_t)}]}
\]

Equation (13) renders an alternative specification to (2) and shows how risk-free rates of longer maturity affect the single factor C-CAPM with the interaction of the long-term risk-free rate scaling consumption growth over the corresponding period and affecting risk aversion. In turn, defining \( R_{t+1,t+1+s}^f \equiv R_{s,t}^f \) for notational simplicity and adopting the standard constant relative risk aversion parametrization, which implies that \( \frac{u'(C_{t+1+s})}{u'(C_t)} \approx 1 - \gamma \Delta^s \ln C_t \), we get that:

\[
\alpha^s = \frac{Cov[R_{s,t}^f (1 - \gamma \Delta^s \ln C_t), R_{t+1,t+1+s}^f]}{E[R_{s,t}^f (1 - \gamma \Delta^s \ln C_t)]}, \quad \beta_j^s = \frac{Cov[R_{s,t}^f \Delta^s \ln C_t, R_{j,t+1}^e]}{Var[R_{s,t}^f \Delta^s \ln C_t]}
\]
\[ \lambda^s = \frac{\gamma_s \text{Var}[R^f_{s,t} \Delta^s \ln C_t]}{E[R^f_{s,t} (1 - \gamma^s \Delta^s \ln C_t)]} \]

The implied risk aversion can be given in terms of the long-term price of risk, \( \lambda^s \), and equals:

\[ \gamma^s = \frac{\lambda^s E[R^f_{s,t}]}{E[R^f_{s,t} \Delta^s \ln C_t]\lambda^s + \text{Var}[R^f_{s,t} \Delta^s \ln C_t]} \]  \hspace{1cm} (14)

which can be larger or smaller than the one implied by (3) depending upon the magnitude of the long-run risk-free rate and the expected value and variance of scaled consumption growth.

Following the spectral approach adopted in section 2, equation (13) can be estimated over the frequency domain as:

\[ E[R^f_{j,t+1}] = \alpha^s_{\omega} + \beta^s_{j,\omega} \lambda^s_{\omega} \]  \hspace{1cm} (15)

where its components, after dropping the time subscript, are given by:

\[ \alpha^s_{\omega} = \frac{f_{R^f_{t} (1 - \gamma^s \Delta^s \ln C), R^f_{t} (\omega)}}{E[R^f_{s} (1 - \gamma^s \Delta^s \ln C)]}, \beta^s_{j,\omega} = G_{R^f_{t}, R^f_{s} \Delta^s \ln C (\omega)}, \lambda^s_{\omega} = \frac{\gamma^s_{\omega} f_{R^f_{t} \Delta^s \ln C, R^f_{s} (\Delta^s \ln C)}(\omega)}{E[R^f_{s} (1 - \gamma^s \Delta^s \ln C)]} \]  \hspace{1cm} (16)

and the coefficient of ‘pseudo’ risk aversion is now given by:

\[ \gamma^s_{\omega} = \frac{\lambda^s_{\omega} E[R^f_{s}]}{E[R^f_{s} \Delta^s \ln C] \lambda^s_{\omega} + f_{R^f_{t} \Delta^s \ln C, R^f_{s} (\Delta^s \ln C)}(\omega)} \]  \hspace{1cm} (17)

which is the analog of (9) when long-term risk-free rates are taken into account.

6.2. Empirical results with long-term risk-free rates

To estimate equation (15) we use data on long-term risk-free interest rates. Since data for each maturity, \( s \), are not readily available to match our consumption and return series, we employ risk-free interest rates with maturities of 1, 3, 5 and 10 years starting in 1953:Q2.\(^{22}\)

Risk-free interest rates are made real by employing as a measure of inflation the quarter-to-quarter change in the chain weighted price deflator, spliced across periods, produced by the Bureau of Economic Analysis. In this respect, we proxy expected interest rates and expected inflation with their realized counterparts over the holding period of the corresponding risk-free asset.

\(^{22}\)The series codes are GS1, GS3,GS5 and GS10 and are available from the Board of Governors of the Federal Reserve System (http://www.research.stlouisfed.org/fred2/).
Before presenting the main empirical results, we briefly discuss the spectral properties of the data. The first row of Figure 6 presents the log-spectra of 1, 3, 5, and 10-year consumption growth for all frequencies (stated as a fraction of \( \pi \)). As expected, the volatility of consumption growth at any horizon increases sharply for lower frequencies and, given the properties of consumption growth over the time domain, the low-frequency variability of consumption growth is amplified when the time horizon increases. Again, the relative concentration of fluctuations in low frequencies is an indication of short-term correlation in consumption growth, which drives the estimates for the coefficients of risk aversion. The second row of Figure 6 plots the estimated coherencies between the long-term returns and the corresponding measures of consumption growth and shows that the relationship remains fairly stable over the whole frequency domain for all four horizons considered. The third row of Figure 6 plots the respective estimated gains over the frequency domain and, as can be readily seen, as the horizon of returns and the corresponding consumption growth rates increase, the gains for higher frequencies are substantially lower.

Table 4 presents the estimates for the four maturities considered. The evidence from the 1-year interest rates (Panel A) replicates the usual failure of the C-CAPM; the high frequency of consumption risk explains only a small fraction of the variation in returns and is associated with a significant equity premium of the magnitude of 3.4% per quarter, whereas the coefficient of ‘pseudo’ risk aversion is found to be 79.7 and is significant. For the 16-quarter horizon the coefficient of risk equity premium falls to 2%, but the model is overall unable to explain the cross-section of returns and the coefficient of risk aversion is statistically equal to zero. As we move to lower frequencies, the picture changes starkly. For the 32-quarter horizon, the performance of the model improves dramatically, the equity premium is negligible, and the coefficient of ‘pseudo’ risk aversion is estimated at 4.5 with a small standard error. The picture is further improved at the zero frequency, where the model explains 96.4% of the variability in returns with a zero equity premium and a significant coefficient of ‘pseudo’ risk aversion found to be as low as 3.4.

A similar picture emerges from other risk-free rates of long-term maturities (Panels B to D of Table 5). In all cases, the model fails at high frequencies, but its performance is consistent with the C-CAPM at lower frequencies. The coefficient of ‘pseudo’ risk aversion attains plausible
values for horizons above 16-quarters for the 3 and 5-year interest rates, whereas the specification with the 10-year risk free rate produces plausible results with the 16-quarter horizon as well. Notably, when the 10-year rate is used the model yields a coefficient of ‘pseudo’ risk aversion for the zero frequency (infinite horizon) that is in the vicinity of unity. These patterns are corroborated by Figure 8, in which the average realized and fitted returns from the various risk-free rates are found to be closely aligned.

7. Conclusions

In this paper we re-evaluated the C-CAPM by adopting a spectral approach to measure the covariance of an asset’s return with consumption growth over the frequency domain and its impact on expected stock returns. We established that when lower frequencies of consumption risk are considered the validity of the C-CAPM is restored. For low frequencies the C-CAPM can explain almost entirely the cross-sectional variation of expected returns accompanied by a decrease in the equity premium.

Recently, there have been some attempts to bring together longer-term consumption dynamics with theoretical explanations. Panageas and Yu (2006) claim that over the short run, consumption growth is dominated by small frequent shocks, while unpredicted and large technological innovations, which are embodied in the capital stock, prevail in the long run. The authors then show that this framework implies that consumption growth over the long run can reveal information about the degree to which the economy has absorbed a major technological shock. Malloy et al. (2006) show that in a model with recursive preferences the covariance of returns with long-run consumption growth of households who bear stock market risk captures the cross-sectional variation of average stock returns better than the covariance of returns with long-run aggregate or non-stockholder consumption growth. Thus, the question on why consumption takes so long to adjust to news in stock returns and what the underlying shocks driving stock returns and consumption are remains open and offers a promising route for further research.
References


Table 1. Expected excess returns and consumption risk frequencies

Panel A

<table>
<thead>
<tr>
<th>Frequency (Quarters)</th>
<th>R-sq(adj)</th>
<th>$\lambda_\omega$</th>
<th>standard error</th>
<th>'pseudo' risk aversion</th>
<th>standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (2.000)</td>
<td>-2.728</td>
<td>0.300</td>
<td>0.043</td>
<td>71.002</td>
<td>0.581</td>
</tr>
<tr>
<td>15/16 (2.133)</td>
<td>-4.460</td>
<td>0.766</td>
<td>0.132</td>
<td>69.735</td>
<td>0.888</td>
</tr>
<tr>
<td>7/8 (2.286)</td>
<td>-1.931</td>
<td>0.971</td>
<td>0.094</td>
<td>68.572</td>
<td>0.594</td>
</tr>
<tr>
<td>13/16 (2.462)</td>
<td>-3.428</td>
<td>0.785</td>
<td>0.088</td>
<td>46.676</td>
<td>0.362</td>
</tr>
<tr>
<td>3/4 (2.667)</td>
<td>-5.218</td>
<td>1.061</td>
<td>0.169</td>
<td>48.650</td>
<td>0.229</td>
</tr>
<tr>
<td>5/8 (3.200)</td>
<td>-1.325</td>
<td>0.319</td>
<td>0.030</td>
<td>33.314</td>
<td>0.344</td>
</tr>
<tr>
<td>1/2 (4.000)</td>
<td>-0.298</td>
<td>0.387</td>
<td>0.019</td>
<td>36.222</td>
<td>0.057</td>
</tr>
<tr>
<td>3/8 (5.333)</td>
<td>-2.076</td>
<td>0.342</td>
<td>0.030</td>
<td>27.510</td>
<td>0.186</td>
</tr>
<tr>
<td>1/4 (8.000)</td>
<td>-3.160</td>
<td>0.365</td>
<td>0.044</td>
<td>17.857</td>
<td>0.073</td>
</tr>
<tr>
<td>3/16 (10.667)</td>
<td>-1.401</td>
<td>0.434</td>
<td>0.041</td>
<td>14.095</td>
<td>0.061</td>
</tr>
<tr>
<td>1/8 (16.000)</td>
<td>-0.443</td>
<td>0.304</td>
<td>0.023</td>
<td>8.822</td>
<td>0.031</td>
</tr>
<tr>
<td>1/16 (32.000)</td>
<td>0.660</td>
<td>0.671</td>
<td>0.021</td>
<td>4.621</td>
<td>0.002</td>
</tr>
<tr>
<td>0 (inf)</td>
<td>0.981</td>
<td>0.677</td>
<td>0.004</td>
<td>4.297</td>
<td>0.002</td>
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</table>

Panel B

<table>
<thead>
<tr>
<th>Frequency (Quarters)</th>
<th>R-sq(adj)</th>
<th>Equity premium</th>
<th>standard error</th>
<th>$\lambda_\omega$</th>
<th>standard error</th>
<th>'pseudo' risk aversion</th>
<th>standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (2.000)</td>
<td>-0.026</td>
<td>2.322</td>
<td>0.292</td>
<td>0.024</td>
<td>0.046</td>
<td>42.463</td>
<td>36.590</td>
</tr>
<tr>
<td>15/16 (2.133)</td>
<td>0.257</td>
<td>3.210</td>
<td>0.300</td>
<td>-0.281</td>
<td>0.086</td>
<td>96.259</td>
<td>8.239</td>
</tr>
<tr>
<td>7/8 (2.286)</td>
<td>0.006</td>
<td>2.917</td>
<td>0.325</td>
<td>-0.186</td>
<td>0.152</td>
<td>154.260</td>
<td>131.933</td>
</tr>
<tr>
<td>13/16 (2.462)</td>
<td>0.042</td>
<td>2.895</td>
<td>0.392</td>
<td>-0.156</td>
<td>0.130</td>
<td>80.082</td>
<td>40.507</td>
</tr>
<tr>
<td>3/4 (2.667)</td>
<td>-0.030</td>
<td>2.593</td>
<td>0.235</td>
<td>-0.064</td>
<td>0.101</td>
<td>101.154</td>
<td>161.341</td>
</tr>
<tr>
<td>5/8 (3.200)</td>
<td>-0.043</td>
<td>2.528</td>
<td>0.554</td>
<td>-0.006</td>
<td>0.078</td>
<td>-6.818</td>
<td>103.596</td>
</tr>
<tr>
<td>1/2 (4.000)</td>
<td>-0.003</td>
<td>1.864</td>
<td>0.467</td>
<td>0.099</td>
<td>0.072</td>
<td>33.174</td>
<td>2.724</td>
</tr>
<tr>
<td>3/8 (5.333)</td>
<td>0.242</td>
<td>3.715</td>
<td>0.605</td>
<td>-0.186</td>
<td>0.078</td>
<td>35.221</td>
<td>2.675</td>
</tr>
<tr>
<td>1/4 (8.000)</td>
<td>0.121</td>
<td>3.125</td>
<td>0.436</td>
<td>-0.110</td>
<td>0.061</td>
<td>20.935</td>
<td>1.526</td>
</tr>
<tr>
<td>3/16 (10.667)</td>
<td>0.022</td>
<td>3.113</td>
<td>0.625</td>
<td>-0.117</td>
<td>0.107</td>
<td>17.991</td>
<td>3.572</td>
</tr>
<tr>
<td>1/8 (16.000)</td>
<td>-0.032</td>
<td>2.159</td>
<td>0.790</td>
<td>0.041</td>
<td>0.105</td>
<td>6.807</td>
<td>4.592</td>
</tr>
<tr>
<td>1/16 (32.000)</td>
<td>0.655</td>
<td>-0.344</td>
<td>0.330</td>
<td>0.761</td>
<td>0.084</td>
<td>4.629</td>
<td>0.007</td>
</tr>
<tr>
<td>0 (inf)</td>
<td>0.986</td>
<td>-0.201</td>
<td>0.080</td>
<td>0.729</td>
<td>0.021</td>
<td>4.323</td>
<td>0.010</td>
</tr>
</tbody>
</table>

Notes:
1) Frequency is expressed as a fraction of $\pi$.
2) See the text for the definition of $\lambda_\omega$ and $\gamma_\omega$.
3) Newey-West heteroskedasticity and autocorrelation corrected standard errors.
4) The equity premium, the price of risk, $\lambda_\omega$, and their standard errors are scaled by 10^2.
### Table 2.

Expected excess returns and consumption risk frequencies: Robustness tests

<table>
<thead>
<tr>
<th>Frequency (Quarters)</th>
<th>R-sq(adj)</th>
<th>Equity premium</th>
<th>standard error</th>
<th>$\lambda_\omega$</th>
<th>standard error</th>
<th>‘pseudo’ risk aversion</th>
<th>standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Original Fama-French start date (1963:03)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (2)</td>
<td>0.296</td>
<td>1.696</td>
<td>0.172</td>
<td>0.051</td>
<td>0.017</td>
<td>60.739</td>
<td>3.902</td>
</tr>
<tr>
<td>1/2 (4)</td>
<td>-0.008</td>
<td>2.808</td>
<td>0.887</td>
<td>-0.058</td>
<td>0.079</td>
<td>45.227</td>
<td>12.923</td>
</tr>
<tr>
<td>1/4 (8)</td>
<td>0.314</td>
<td>3.282</td>
<td>0.328</td>
<td>-0.188</td>
<td>0.038</td>
<td>19.945</td>
<td>0.317</td>
</tr>
<tr>
<td>1/8 (16)</td>
<td>-0.016</td>
<td>1.745</td>
<td>0.718</td>
<td>0.075</td>
<td>0.110</td>
<td>7.468</td>
<td>2.107</td>
</tr>
<tr>
<td>1/16 (32)</td>
<td>0.671</td>
<td>-0.531</td>
<td>0.201</td>
<td>0.899</td>
<td>0.070</td>
<td>4.612</td>
<td>0.006</td>
</tr>
<tr>
<td>0 (inf)</td>
<td>0.995</td>
<td>-0.211</td>
<td>0.046</td>
<td>0.745</td>
<td>0.014</td>
<td>4.269</td>
<td>0.007</td>
</tr>
<tr>
<td><strong>B. Total consumption</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (2)</td>
<td>0.016</td>
<td>1.795</td>
<td>0.602</td>
<td>0.077</td>
<td>0.074</td>
<td>23.540</td>
<td>4.031</td>
</tr>
<tr>
<td>1/2 (4)</td>
<td>0.000</td>
<td>1.976</td>
<td>0.465</td>
<td>0.049</td>
<td>0.048</td>
<td>23.674</td>
<td>4.005</td>
</tr>
<tr>
<td>1/4 (8)</td>
<td>0.098</td>
<td>1.206</td>
<td>0.552</td>
<td>0.209</td>
<td>0.101</td>
<td>9.721</td>
<td>0.287</td>
</tr>
<tr>
<td>1/8 (16)</td>
<td>-0.033</td>
<td>2.200</td>
<td>0.718</td>
<td>0.039</td>
<td>0.104</td>
<td>5.367</td>
<td>3.551</td>
</tr>
<tr>
<td>1/16 (32)</td>
<td>0.712</td>
<td>0.321</td>
<td>0.252</td>
<td>0.774</td>
<td>0.085</td>
<td>3.545</td>
<td>0.006</td>
</tr>
<tr>
<td>0 (inf)</td>
<td>0.989</td>
<td>-0.049</td>
<td>0.075</td>
<td>0.915</td>
<td>0.024</td>
<td>3.311</td>
<td>0.007</td>
</tr>
<tr>
<td><strong>C. Long-horizon returns</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (2)</td>
<td>-0.006</td>
<td>50.567</td>
<td>7.801</td>
<td>-1.064</td>
<td>1.308</td>
<td>76.719</td>
<td>1.777</td>
</tr>
<tr>
<td>1/2 (4)</td>
<td>0.022</td>
<td>40.704</td>
<td>3.327</td>
<td>1.615</td>
<td>1.089</td>
<td>37.100</td>
<td>0.205</td>
</tr>
<tr>
<td>1/4 (8)</td>
<td>0.421</td>
<td>52.874</td>
<td>3.045</td>
<td>-0.923</td>
<td>0.230</td>
<td>18.768</td>
<td>0.071</td>
</tr>
<tr>
<td>1/8 (16)</td>
<td>0.272</td>
<td>20.540</td>
<td>7.091</td>
<td>1.038</td>
<td>0.335</td>
<td>9.112</td>
<td>0.044</td>
</tr>
<tr>
<td>1/16 (32)</td>
<td>0.422</td>
<td>1.295</td>
<td>9.299</td>
<td>1.215</td>
<td>0.258</td>
<td>4.653</td>
<td>0.008</td>
</tr>
<tr>
<td>0 (inf)</td>
<td>0.979</td>
<td>1.657</td>
<td>1.760</td>
<td>0.689</td>
<td>0.026</td>
<td>4.306</td>
<td>0.013</td>
</tr>
</tbody>
</table>

*Notes: See Table 1.*
### Table 3.

Expected excess returns and consumption risk frequencies: Alternative portfolios

<table>
<thead>
<tr>
<th>Frequency</th>
<th>R-sq(adj)</th>
<th>Equity premium</th>
<th>standard error</th>
<th>$\lambda_\omega$</th>
<th>standard error</th>
<th>'pseudo' risk aversion</th>
<th>standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Equally weighted portfolios</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (2)</td>
<td>0.092</td>
<td>2.146</td>
<td>0.259</td>
<td>0.061</td>
<td>0.038</td>
<td>58.129</td>
<td>8.248</td>
</tr>
<tr>
<td>1/2 (4)</td>
<td>-0.025</td>
<td>3.185</td>
<td>1.060</td>
<td>-0.086</td>
<td>0.146</td>
<td>43.895</td>
<td>13.017</td>
</tr>
<tr>
<td>1/4 (8)</td>
<td>0.034</td>
<td>3.177</td>
<td>0.403</td>
<td>-0.093</td>
<td>0.059</td>
<td>21.468</td>
<td>2.216</td>
</tr>
<tr>
<td>1/8 (16)</td>
<td>0.091</td>
<td>1.434</td>
<td>0.664</td>
<td>0.141</td>
<td>0.087</td>
<td>8.373</td>
<td>0.491</td>
</tr>
<tr>
<td>1/16 (32)</td>
<td>0.733</td>
<td>0.063</td>
<td>0.260</td>
<td>0.711</td>
<td>0.082</td>
<td>4.625</td>
<td>0.007</td>
</tr>
<tr>
<td>0 (inf)</td>
<td>0.992</td>
<td>-0.081</td>
<td>0.045</td>
<td>0.704</td>
<td>0.013</td>
<td>4.311</td>
<td>0.006</td>
</tr>
<tr>
<td>B. 10 size, 10 B/M and 10 D/P portfolios</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (2)</td>
<td>0.132</td>
<td>1.911</td>
<td>0.116</td>
<td>0.060</td>
<td>0.017</td>
<td>57.694</td>
<td>3.898</td>
</tr>
<tr>
<td>1/2 (4)</td>
<td>0.110</td>
<td>1.665</td>
<td>0.180</td>
<td>0.088</td>
<td>0.031</td>
<td>25.304</td>
<td>2.874</td>
</tr>
<tr>
<td>1/4 (8)</td>
<td>0.052</td>
<td>1.888</td>
<td>0.158</td>
<td>0.076</td>
<td>0.035</td>
<td>20.240</td>
<td>1.723</td>
</tr>
<tr>
<td>1/8 (16)</td>
<td>0.198</td>
<td>1.367</td>
<td>0.246</td>
<td>0.118</td>
<td>0.035</td>
<td>8.225</td>
<td>0.267</td>
</tr>
<tr>
<td>1/16 (32)</td>
<td>0.507</td>
<td>0.835</td>
<td>0.174</td>
<td>0.377</td>
<td>0.045</td>
<td>4.570</td>
<td>0.014</td>
</tr>
<tr>
<td>0 (inf)</td>
<td>0.939</td>
<td>-0.032</td>
<td>0.105</td>
<td>0.681</td>
<td>0.031</td>
<td>4.300</td>
<td>0.016</td>
</tr>
<tr>
<td>C. 20 risk-sorted portfolios</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (2)</td>
<td>-0.056</td>
<td>1.939</td>
<td>0.100</td>
<td>0.000</td>
<td>0.014</td>
<td>-1.509</td>
<td>60.938</td>
</tr>
<tr>
<td>1/2 (4)</td>
<td>0.024</td>
<td>2.195</td>
<td>0.196</td>
<td>-0.045</td>
<td>0.031</td>
<td>52.154</td>
<td>14.198</td>
</tr>
<tr>
<td>1/4 (8)</td>
<td>-0.051</td>
<td>1.978</td>
<td>0.138</td>
<td>-0.007</td>
<td>0.022</td>
<td>-24.615</td>
<td>168.524</td>
</tr>
<tr>
<td>1/8 (16)</td>
<td>-0.046</td>
<td>1.999</td>
<td>0.169</td>
<td>-0.008</td>
<td>0.019</td>
<td>-12.211</td>
<td>64.929</td>
</tr>
<tr>
<td>1/16 (32)</td>
<td>-0.054</td>
<td>1.912</td>
<td>0.157</td>
<td>0.007</td>
<td>0.036</td>
<td>1.908</td>
<td>5.983</td>
</tr>
<tr>
<td>0 (inf)</td>
<td>0.818</td>
<td>-0.299</td>
<td>0.272</td>
<td>0.751</td>
<td>0.092</td>
<td>4.333</td>
<td>0.040</td>
</tr>
<tr>
<td>D. 34 industry portfolios</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 (2)</td>
<td>-0.019</td>
<td>1.936</td>
<td>0.156</td>
<td>0.010</td>
<td>0.021</td>
<td>27.162</td>
<td>36.261</td>
</tr>
<tr>
<td>1/2 (4)</td>
<td>0.003</td>
<td>2.161</td>
<td>0.194</td>
<td>-0.030</td>
<td>0.032</td>
<td>52.657</td>
<td>43.332</td>
</tr>
<tr>
<td>1/4 (8)</td>
<td>0.062</td>
<td>1.602</td>
<td>0.210</td>
<td>0.059</td>
<td>0.027</td>
<td>20.054</td>
<td>3.040</td>
</tr>
<tr>
<td>1/8 (16)</td>
<td>-0.030</td>
<td>1.957</td>
<td>0.378</td>
<td>0.005</td>
<td>0.048</td>
<td>2.306</td>
<td>17.053</td>
</tr>
<tr>
<td>1/16 (32)</td>
<td>0.066</td>
<td>1.731</td>
<td>0.172</td>
<td>0.081</td>
<td>0.040</td>
<td>4.180</td>
<td>0.223</td>
</tr>
<tr>
<td>0 (inf)</td>
<td>0.853</td>
<td>0.247</td>
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<td>0.582</td>
<td>0.051</td>
<td>4.240</td>
<td>0.036</td>
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</tbody>
</table>

Notes: See Table 1.
Table 4.
Expected excess returns and consumption risk frequencies: Long-term interest rates

<table>
<thead>
<tr>
<th>Frequency (Quarters)</th>
<th>R-sq(adj)</th>
<th>Equity premium standard error</th>
<th>λω standard error</th>
<th>'pseudo' risk aversion standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. 1-year interest rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(2)</td>
<td>0.143</td>
<td>3.378</td>
<td>0.561</td>
</tr>
<tr>
<td>1/2</td>
<td>(4)</td>
<td>0.156</td>
<td>1.282</td>
<td>0.557</td>
</tr>
<tr>
<td>1/4</td>
<td>(8)</td>
<td>0.103</td>
<td>3.241</td>
<td>0.539</td>
</tr>
<tr>
<td>1/8</td>
<td>(16)</td>
<td>-0.017</td>
<td>2.023</td>
<td>0.614</td>
</tr>
<tr>
<td>1/16</td>
<td>(32)</td>
<td>0.730</td>
<td>-0.176</td>
<td>0.340</td>
</tr>
<tr>
<td>0</td>
<td>(inf)</td>
<td>0.964</td>
<td>0.026</td>
<td>0.123</td>
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<tr>
<td><strong>B. 3-year interest rate</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(2)</td>
<td>0.127</td>
<td>2.904</td>
<td>0.325</td>
</tr>
<tr>
<td>1/2</td>
<td>(4)</td>
<td>0.059</td>
<td>1.614</td>
<td>0.639</td>
</tr>
<tr>
<td>1/4</td>
<td>(8)</td>
<td>0.499</td>
<td>3.549</td>
<td>0.307</td>
</tr>
<tr>
<td>1/8</td>
<td>(16)</td>
<td>0.215</td>
<td>0.832</td>
<td>0.510</td>
</tr>
<tr>
<td>1/16</td>
<td>(32)</td>
<td>0.659</td>
<td>1.015</td>
<td>0.154</td>
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<tr>
<td>0</td>
<td>(inf)</td>
<td>0.968</td>
<td>0.195</td>
<td>0.109</td>
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<tr>
<td><strong>C. 5-year interest rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(2)</td>
<td>0.008</td>
<td>2.318</td>
<td>0.214</td>
</tr>
<tr>
<td>1/2</td>
<td>(4)</td>
<td>0.008</td>
<td>2.153</td>
<td>0.350</td>
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<tr>
<td>1/4</td>
<td>(8)</td>
<td>0.215</td>
<td>3.557</td>
<td>0.445</td>
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<tr>
<td>1/8</td>
<td>(16)</td>
<td>0.266</td>
<td>0.052</td>
<td>0.774</td>
</tr>
<tr>
<td>1/16</td>
<td>(32)</td>
<td>0.760</td>
<td>1.028</td>
<td>0.161</td>
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<tr>
<td>0</td>
<td>(inf)</td>
<td>0.939</td>
<td>0.276</td>
<td>0.170</td>
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<td><strong>D. 10-year interest rate</strong></td>
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<td></td>
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<tr>
<td>1</td>
<td>(2)</td>
<td>0.021</td>
<td>2.294</td>
<td>0.191</td>
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<tr>
<td>1/2</td>
<td>(4)</td>
<td>0.011</td>
<td>2.121</td>
<td>0.334</td>
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<tr>
<td>1/4</td>
<td>(8)</td>
<td>0.295</td>
<td>3.354</td>
<td>0.293</td>
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<tr>
<td>1/8</td>
<td>(16)</td>
<td>0.072</td>
<td>1.179</td>
<td>0.590</td>
</tr>
<tr>
<td>1/16</td>
<td>(32)</td>
<td>0.442</td>
<td>-0.282</td>
<td>0.288</td>
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<tr>
<td>0</td>
<td>(inf)</td>
<td>0.929</td>
<td>0.052</td>
<td>0.182</td>
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</table>

Notes: See Table 1.
Figure 1A. Spectrum of non-durables consumption growth

Figure 1B. Spectrum of excess returns

Figure 1C. Coherency over the Spectrum: Excess returns and non-durables consumption growth

Notes: 95% confidence intervals in dashed lines.
Figure 2. Average returns and betas

Horizon = 2 quarters

Horizon = 8 quarters

Horizon = 32 quarters

Infinite Horizon

Average returns

Consumption betas (gains)
Figure 3. Fitted and average returns

Horizon = 2 quarters

Horizon = 8 quarters

Horizon = 32 quarters

Infinite Horizon

Average realized returns
Figure 4. Fitted and average returns (alternative specifications, infinite horizon)

Fama-French start date

Total Consumption

Fitted returns

Long-horizon returns

Average realized returns
Figure 5. Fitted and average returns (alternative portfolios, infinite horizon)

Equally-weighted portfolios

20 risk-sorted portfolios

10 size, 10 B/M & 10 D/P portfolios

34 industry portfolios

Average realized returns
Figure 6. Spectral properties of C-CAPM and the term structure of interest rates

Log-spectrum of scaled consumption growth

Coherency of scaled consumption growth and returns

Gain of scaled consumption growth and returns
Figure 7. Fitted and average returns (term structure, infinite horizon)

Fitted returns

Average realized returns